

Screening of Possibly Incompetent Agents

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Abstract

Accepting a contract with a high incentive for performance is normally interpreted as a signal of high ability. However, an extremely high self-assessment may be an incompetent forecast by an incompetent worker. The authors do not want an investment advisor who expects nearly riskfree returns of 50%/year! We study separating equilibria in a model in which optimistic forecasters have low ability. In one type of equilibrium, a low incentive screens out the incompetent agents. In another, agents are wealthy enough for the principal to select the incompetent agents who cover the downside (as in a vanity press).

1 Introduction

Models in which agents are selected (or self-select) based on ability typically assume that beliefs come from common priors.¹ In these models, incentive pay tends to attract agents of higher ability and screens out weaker candidates. For example, in the context of investment portfolio management, Bhattacharya and Pfleiderer (1985) show that, given common priors about managers' abilities, more competent managers self-select into positions with higher pay-for-performance. In practice, however, there are no contests where managers are judged to be best if they take higher risk. We show that allowing agents to have different priors about their abilities permits a simple and plausible explanation for why this does not happen. Agents who are relatively pessimistic about their ability are likely to avoid jobs with any incentive pay, while agents who are very optimistic will self-select themselves for jobs with big incentive pay. This may be a problem for employers expecting to attract good workers, especially when documented ability is scarce and self-assessed ability is great (as in the portfolio management industry). In this environment, instead of searching for competent managers, employers may prefer to hire overly optimistic agents, but only if these agents have significant assets to pledge. An agent with a strong belief in own ability will be willing to guarantee the performance with personal wealth, and it will not matter if the employer thinks the agent cannot deliver.² A good example of this is a vanity press that charges authors to publish their work.³ When such guarantees are not feasible (because the potential employee is not rich enough, the typical situation in

¹See, for example, Spence (1973), Bhattacharya and Pfleiderer (1985), Koszegi and Li (2008).

A recent survey of screening and signalling models is provided in Riley (2001)

²We formally develop this point in section 2.

³Some authors who use a vanity press do not care about sales and simply want to produce a beautiful book, possibly just to give their friends. These are not the people in our models.

money management), employers may have to offer inefficiently low performance incentives to screen out overly optimistic employees.

Our main model has a principal who seeks to hire an agent to fill a single position. We study equilibria in a pure screening setting (hidden exogenous type but no hidden effort) with two types of potential manager. From the perspective of the principal, one type is a good manager and is only slightly over-confident,⁴ while the other type is not a good manager but is extremely over-confident. Except in welfare discussions, it does not matter which agent, if any, has “correct” beliefs. We show that, to attract the more competent agents, the principal offers a wage schedule with little incentive compensation, making it unattractive to the incompetent but extremely confident agents. The principal, however, finds attracting incompetent agents more profitable if the agents have large pledgeable wealth (implying that the lower bound on wage is negative and large in absolute value). In this case, the principal offers a contract with a big embedded bet on output. Such a bet is attractive only to the extremely confident, incompetent type. We show that our results are robust to adding pessimistic and incompetent agents, and extend to the setting with multiple positions.

It is in principle possible to test our model empirically by studying the link between incentives in compensation and portfolio performance. As for most information models, however, it is difficult to construct a robust test with the available data. For example, while some studies find that stronger monetary incentives correspond to better performance (for example, Khorana, Servas, and Wedge (2007)), it is hard to tell from the evidence whether the positive relationship is due to effort or ability, and it is especially

⁴Although, as we show in section 4, our results still hold if the good manager is under-confident

difficult to link the results to the managers' perceptions of their ability. A number of empirical measures of managerial overconfidence are offered in Malmendier and Tate (2005); evidence of managerial overconfidence is also reported in Ben-David, Graham, and Harvey (2007). They, however, do not look at the link between their measures and CEO compensation packages, and that may be one place to look for evidence that can be used to test our model.

Our paper is part of the growing literature which studies agents who have different prior beliefs. Santos-Pinto and Sobel (2005) illustrate where differences in beliefs might come from. Consistency between individual rationality and differences in beliefs, is also discussed in Van den Steen (2004). Van den Steen (2004) also shows that agents with different priors tend to overestimate their ability to control the outcome, achieve success, and outperform others. Van den Steen (2005) focuses on team structure and argues that agents with similar priors are more likely to self-select into the same firm. The effect of managerial overconfidence (which can be viewed as disagreement with the principal) on the firm investment policy and manager's welfare is studied in Gervais, Heaton, and Odean (2007). Adrian and Westerfield (2008) use a dynamic setting to analyze how disagreement between the agent and the principal impacts the optimal risk sharing. The possibility that introducing agents with biased ability estimates may significantly affect the optimal compensation structure in a screening model is noted by Dybvig, Farnsworth, and Carpenter (2004), but they do not develop this point. We contribute to the literature by studying self-selection in the presence of fundamental disagreement about agents' abilities.

Manove and Padilla (1999) present analysis that is closely related to ours, although it is not presented in terms of heterogeneous prior beliefs. Manove and Padilla consider a set-

ting where the principal is a bank, and some of the agents (borrowers) are assumed to be irrationally overoptimistic about their investment opportunities. Similar to us, they show that the principal may choose to contract with the less productive overoptimistic type despite the type's lower productivity. There are several differences in our analyses. Most importantly, our paper does not take a stand on whose beliefs represent the true probabilities, and thus produces a different welfare analysis. Additionally, our model assumes that the principal disagrees with both agent types, and that the agents receive no rents, while Manove and Padilla assume that the principal agrees with one type (which makes the structure of the contract with this type uninteresting) and that the agents receive all the rents. Finally, we solve for the optimal contract, while Manove and Padilla restrict their attention to debt-type contracts. These differences result in different equilibrium contracts produced by the two models.

This paper is organized as follows. Section 2 describes the model and derives the equilibria. Section 4 shows that our model results are robust to various extensions, and section 5 concludes. The formal proofs are in the Appendix.

2 Model

There is a risk-neutral principal who wishes to hire one manager. There are two types of agents who can become managers: type o (optimistic, overconfident) and type c (conservative, competent). The type of each agent is known to the agent but not to the principal. Let N be the finite total number of agents, let π_o be the proportion of agents that are type o , and let $\pi_c = 1 - \pi_o$ be the proportion of agents that are type c . All agents are risk-neutral over nonnegative wealth. The firm's output equals the manager's ability

$a \in \{a_l, a_h\}$, where $a_h > a_l$. The agents' true ability is unknown to both the principal and the agents. Type o (resp. c) agents believe that their ability is high with probability q_o (resp. q_c), while the principal believes that their ability is high with probability f_o (resp. f_c). Beliefs about probabilities for various types are public knowledge,⁵ (although agent types are private information) and we make no assumptions about the true distribution of the agents' abilities. For example, the equilibrium is the same whether type o agents have unrealistically optimistic expectations or the principal just doesn't appreciate how good they are. When we discuss the model results, however, we often take the view of the principal, as it seems natural and helps the exposition.

We assume that $f_o < f_c < q_c < q_o$, which implies that both types of agents place a larger probability on their ability to be high than the principal does, and the principal believes that overly confident agents are less likely to have high ability. This is the interesting case, although the intuition is similar when the agents are less optimistic than the principal, as we discuss in section 4. In that section, we also show that adding pessimistic ($q_p < q_c$) and incompetent ($f_p < f_o$) agents with a reasonably high reservation utility would not change the equilibrium because such agents would not accept the optimal contracts we describe, and the principal is happy about that.

The principal advertises the managerial position with the wage schedule $w = (w_l, w_h)$, where compensation is w_l if the output is a_l and w_h if the output is a_h . We require the wage to be nondecreasing in the output: $w_h \geq w_l$ (a decreasing wage would give agents an incentive to destroy part of the output to receive a higher wage). All agents choose simultaneously whether to apply. We denote the decision of a type o (resp. c) agent to

⁵In the model, the candidate managers are price-takers and thus do not need to know other agents' abilities and beliefs.

apply for wage schedule w using function $m_o(w)$ (resp. $m_c(w)$) which takes on the value of 1 if the agent applies and 0 if the agent does not. Note that functions $m_o(w)$ and $m_c(w)$ are defined for any feasible wage w (in an extensive form game, agents specify responses to both on-equilibrium and off-equilibrium actions of other agents). If only one agent applies, the agent is hired. If several agents apply, the principal randomly picks an agent. If no agent applies, no one is hired and the principal receives a normalized zero payoff. We employ the solution concept of *subgame perfect Bayesian Nash equilibrium* in symmetric pure strategies, where by symmetric we mean agents of the same type play the same strategy. In this game, the only proper subgames are the whole game and the subgames starting after the wage schedule is locked in, when the agents choose whether to apply.

We assume that each agent is endowed with the same initial wealth $W \geq 0$ (because we are not studying screening on initial wealth), so we impose the following limited resources constraint: $w_h, w_l \geq -W$. All type o (resp. c) agents not hired by the principal receive expected wage u_o (resp. u_c) in the outside market. Therefore, given other agents' application strategies, the agent of type $j \in \{o, c\}$ maximizes expectation of utility equal to

$$(1) \quad \begin{cases} q_j w_h + (1 - q_j) w_l & \text{if hired;} \\ u_j & \text{if not hired.} \end{cases}$$

Given there are finitely many agents, the probability of getting hired is 0 if the agent does not apply ($m_j(w) = 0$) and positive if the agent applies ($m_j(w) = 1$), whatever the strategies played by the other agents. Therefore we do not need to do any detailed calculations to show that a strategy is optimal if and only if it satisfies

$$(2) \quad m_j(w) \begin{cases} = 1 & \text{if } q_j w_h + (1 - q_j) w_l > u_j; \\ \in [0, 1] & \text{if } q_j w_h + (1 - q_j) w_l = u_j; \\ = 0 & \text{if } q_j w_h + (1 - q_j) w_l < u_j, \end{cases}$$

for all w . Note that, since we focus on equilibrium in pure strategies, we can restrict attention to $m_j(w) \in \{0, 1\}$. The payoff for the principal from offering w is

$$(3) \quad \begin{cases} 0 & \text{if } m_o(w) = m_c(w) = 0 \\ f_o(a_h - w_h) + (1 - f_o)(a_l - w_l) & \text{if } m_o(w) = 1 \text{ and } m_c(w) = 0; \\ f_c(a_h - w_h) + (1 - f_c)(a_l - w_l) & \text{if } m_o(w) = 0 \text{ and } m_c(w) = 1; \\ \sum_{j=o,c} \pi_j (f_j(a_h - w_h) + (1 - f_j)(a_l - w_l)) & \text{if } m_o(w) = m_c(w) = 1. \end{cases}$$

The following restrictions on the parameter space let us focus on cases that make our economic point.

Assumption 1. *a. The principal believes that all agents are less skilled than they think, and that more optimistic agents (type o) are less skilled than more conservative agents (type c):*

$$f_o < f_c < q_c < q_o.$$

b. Even a low-ability manager produces enough output to make the firm profitable:

$$a_l > u_o.$$

c. Reservation utility of type o is above that of type c:

$$u_o > u_c.$$

d. Reservation utility of type o is not too large:

$$q_c(u_o + W) \leq q_o(u_c + W).$$

Assumption 1b is sufficient to ensure that the principal will hire someone in equilibrium. As we show in the proof of Theorem 1, assumption 1c is a necessary condition for hiring type c in equilibrium, and assumption 1d, which imposes an upper boundary on the

reservation utility u_o , is a necessary condition for hiring type o in equilibrium. When u_o is above this upper boundary imposed by 1d, the model becomes trivial: the principal hires only the less confident type c because this type would be both cheaper and more productive, and the presence of type o becomes irrelevant for both the hiring and the compensation decisions.

2.1 Equilibrium

Given our parameter restrictions specified in Assumption 1, there are two possible types of symmetric separating equilibria and one type of pooling equilibrium.⁶ The following theorem characterizes these equilibria.

Theorem 1. *Given Assumption 1, if there is a separating equilibrium, it is either Equilibrium VP or Equilibrium TS, defined below. If there is a pooling equilibrium, it is Equilibrium NS, also defined below. Equilibrium VP, TS, or NS is realized if it leaves the principal with the maximum payoff of $\max\{\Pi_{VP}, \Pi_{TS}, \Pi_{NS}\}$ (payoffs Π_{VP} , Π_{TS} , and Π_{NS} are defined below).*

Equilibrium VP (Vanity Press): *only the less competent (according to the principal)*

type o agents apply. In this equilibrium, the principal offers wage $w^ = (w_l^*, w_h^*)$*

with

$$(4) \quad \begin{aligned} w_h^* &= \frac{u_o + W(1 - q_o)}{q_o}, \\ w_l^* &= -W. \end{aligned}$$

⁶Absent Assumption 1, there can be a pooling equilibrium in which no agent applies for the position, for example if both have reservation wages that are too high.

The agents' responses are given by (2) with $m_c(w^*) = 0$, $m_o(w^*) = 1$. The principal hires a type o agent and receives the following payoff:

$$(5) \quad \Pi_{vp} = f_o a_h + (1 - f_o) a_l - \frac{f_o}{q_o} u_o + \frac{q_o - f_o}{q_o} W;$$

Equilibrium TS (Talent Screening). Only the more competent (according to the principal) type c agents apply. In this equilibrium, the principal offers wage $w^* = (w_l^*, w_h^*)$ with

$$(6) \quad \begin{aligned} w_h^* &= \frac{(1 - q_c)u_o - (1 - q_o)u_c}{q_o - q_c}, \\ w_l^* &= \frac{q_o u_c - q_c u_o}{q_o - q_c}. \end{aligned}$$

The agents' responses are given by (2) with $m_c(w^*) = 1$, $m_o(w^*) = 0$. The principal hires a type c agent and receives the following payoff:

$$(7) \quad \Pi_{ts} = f_c a_h + (1 - f_c) a_l - \frac{(q_o - f_c)u_c - (q_c - f_c)u_o}{q_o - q_c}.$$

Equilibrium NS (No Screening). Both agent types apply. In this equilibrium, the principal offers wage $w^* = (w_l^*, w_h^*)$ with

$$(8) \quad \begin{aligned} w_h^* &= \frac{u_c + W(1 - q_c)}{q_c}, \\ w_l^* &= -W; \end{aligned}$$

The agents' responses are given by (2) with $m_c(w^*) = m_o(w^*) = 1$. The principal hires a type c with probability π_c and hires type o with probability π_o and receives the following payoff:

$$(9) \quad \Pi_{ns} = f_m a_h + (1 - f_m) a_l - \frac{f_m}{q_c} u_c + \frac{q_c - f_m}{q_c} W$$

where $f_m \equiv f_o \pi_o + f_c \pi_c$ is the probability (according to the principal) that the hired agent has high ability.

Proof: See the Appendix.

Because the problem faced by the principal is linear, the wages that appear in the equilibria are at extreme points. We put enough structure on the model to make sure that both types of separating equilibrium can occur, as illustrated in Figure 1. In this figure, the two solid lines represent the agents' participation constraints. The axes are drawn to start at $(-W, -W)$, and therefore, each line represents the wealth the agent will have at the end including the initial endowment of W . Because the optimistic agent's reservation utility is assumed to be above the more conservative agent's reservation utility, the lines cross above the 45 degree line. The dashed lines in the figure represent the principal's indifference curves. Note that the principal's indifference curves are steeper than those of either agent because the principal in our model has the most conservative beliefs about the agents' abilities. Moreover, the slope of the principal's indifference curve depends on which agent the principal expects to hire. The indifference curve corresponding to hiring type o is the steepest, since the principal is the most skeptical about the ability of type o .

The principal wishes to minimize the expected wage expenditure. The smallest expected compensation package that attracts type o is represented in the figure by the point called "Equilibrium VP". This package requires type o to post wealth W as a guarantee of high outcome. The agent has positive wealth only if the outcome is indeed high. This positive wealth level is just sufficient to provide the agent with reservation utility in expectation. Similarly, the smallest expected compensation package that attracts agent c is represented in the figure by the point called "CO" (stands for "competent only"; this compensation would be used if all agents in the market were competent). This compensation package, however, also attracts type o (it is above type o 's reservation utility line).

For the principal, the cheapest way to attract type c without attracting type o is to offer the compensation package denoted in the figure as “Equilibrium TS”. Technically, given the wage in “Equilibrium TS”, type o is indifferent between applying and not applying. However, it is not an equilibrium for type o to apply: if any type o agents were expected to apply, the principal wishes to deviate and offer a wage that lies above type c ’s reservation line but below type o ’s reservation line. This wage is only marginally more expensive to the principal, and is guaranteed to attract only type c . In this equilibrium, even though type o is not hired, the presence of type o impacts both the equilibrium compensation of type c . In particular, hiring the type c is more expensive when the agent’s type o competitors are more optimistic (q_o is larger) or are willing to work for less (u_o is smaller).⁷ The presence of type o also lowers the performance sensitivity of the equilibrium wage; in the extreme case when the agents have the same reservation utility: $u_o = u_c$, the equilibrium wage is flat: $w_h = w_l = u_c$. In general, the performance sensitivity attracting the competent type c (Equilibrium TS) is below that attracting type o (Equilibrium VP). This contrasts the results in Bhattacharya and Pfleiderer, where more able managers are attracted by higher performance sensitivity. In our model, better managers are more measured forecasters, and are therefore attracted by smaller performance sensitivity; the strongest incentives occur in the pooling equilibrium where no screening takes place. The observations on

⁷This differs from Manove and Padilla where the presence of the overconfident type has no impact on the contract with the less confident type. To facilitate the comparison, it is important to note that we use the type terminology differently. In Manove and Padilla, agents receive either high or low signals about their project quality. The low quality signals are unbiased, while the high quality signals may or may not be biased. The principal knows the probability of the bias, but the agents believe that all signals are unbiased. Even though neither the agents nor the principal can tell whether high quality signal is misleading, Manove and Padilla call overoptimistic only those agents whose high quality signals are misleading. Using our terminology, however, we would refer to all agents with high quality signals as overoptimistic (all agents ignore the possibility that their signal is biased). In Manove and Padilla, presence of agents with high quality signals has no impact on the lending to agents with low quality signals because both the principal and the agent agree about payoffs of those projects.

incentives in compensation are formalized in the following corollary.

Corollary 1. *Let incentive strength in compensation (pay-for-performance sensitivity) be measured as $PPS = (w_h - w_l)/(a_h - a_l)$, and let PPS_i denote the incentive strength of equilibrium wage in equilibrium i . Then, $PPS_{NS} > PPS_{VP} > PPS_{TS}$.*

Proof: From the equilibrium expressions for wages in Theorem 1, incentive strengths in Equilibria NS, VP, and TS are given by, respectively,

$$\begin{aligned} PPS_{NS} &= \frac{u_c + W}{(a_h - a_l)q_c}; \\ PPS_{VP} &= \frac{u_o + W}{(a_h - a_l)q_o}; \\ PPS_{TS} &= \frac{u_o - u_c}{(a_h - a_l)(q_o - q_c)}. \end{aligned}$$

The inequalities $PPS_{NS} > PPS_{VP} > PPS_{TS}$ follow from the above expressions after some algebra using Assumption d.

Theorem 1 describes all potential equilibria in our model. We next discuss which equilibrium will be realized, depending on the model parameters.

Theorem 2. *Equilibrium VP, TS, or NS is realized if it leaves the principal with the maximum payoff of $\max\{\Pi_{VP}, \Pi_{TS}, \Pi_{NS}\}$. Additionally, any subset of $\{VP, TS, NS\}$, can be a complete description of all possible equilibria for some parameter values.*

Proof. Theorem 1 shows that the optimal way to attract type o is by offering wage schedule (4), which results in payoff (5), the optimal way to attract type c is by offering wage schedule (6), which results in payoff (7), and the optimal way to attract both types is by

offering wage schedule (8), which results in payoff (9). By assumption 1, attracting no agents is not beneficial for the principal. Therefore, in equilibrium, the principal chooses out of these three wage schedules the wage schedule (or schedules) that results in the highest payoff.

We show feasibility of any set of equilibria using numerical examples summarized in Table 1. □

Comparing the principal's payoffs Π_{vp} and Π_{ts} given by (5) and (7) respectively suggests that the principal chooses to hire type o (Equilibrium VP occurs) when agents have high initial wealth W . Intuitively, because of type o 's lack of ability ($f_o < f_c$) and the higher reservation utility $u_o > u_c$, hiring a type o agent is desirable only when the value of arbitraging the difference in beliefs is high, which is true when W is large. Being overly optimistic, type o is willing to bet all the cash on high outcome. The other side of this bet is attractive for the principal who believes high outcome is not very likely. For example, suppose that $W = 1$ and let us also assume that $\pi_o = 0.9$, $u_o = 2.1$, $u_c = 2$, $q_o = 0.6$, $q_c = 0.5$, $f_o = 0.1$, $f_c = 0.3$, and $a_h - a_l = 10$. Substituting these parameters into (5) and (7) produces $\Pi_{vp} > \Pi_{ts}$, implying that Equilibrium VP is the unique separating equilibrium, where only the optimistic type o is hired. If, however, we reduce the agents' initial wealth to $W = 0.5$, the assumed parameters lead to $\Pi_{ts} > \Pi_{vp}$, and Equilibrium TS becomes the unique separating equilibrium, where only the competent type c is hired.⁸ Comparing (5) and (7) also suggests that the principal may choose to hire type o even when agents have no initial wealth ($W = 0$) but type o is extremely optimistic ($q_o \gg f_o$).

⁸It can also be verified that there is no pooling equilibrium with the above parameter values for both $W = 0.5$ and $W = 1$.

In this case, the principal views type o as cheap: the required compensation is positive only in the unlikely event of high outcome.

Comparing the principal's payoff Π_{ns} in the pooling equilibrium to the separating equilibrium payoffs Π_{vp} and Π_{ts} suggests that the principal chooses to offer the pooling equilibrium wage when hiring type c is attractive to the principal (for example because f_c is large), and there too few type o agents (π_o is small) to justify screening.

3 Welfare Implications

When agents have different priors, social welfare depends on which prior beliefs are chosen for utility valuation. For example, one may take the view of the principal, and use the principal's beliefs for valuing expected outcomes. This approach is reasonable if we view agents as biased (unrealistically optimistic in our case), and the principal as unbiased. The social planner interested in evaluating the unbiased expected utility of all agents disregards personal biases in agents's perception of their own expected utility. This is the approach taken in Manove and Padilla (1999).⁹ Under this approach, hiring the overoptimistic type results in a dead weight loss, even though, for some parameter values, it is attractive for the principal. Therefore, regulations that discourage hiring overconfident type may be desirable. One example of such regulation, discussed in Manove and Padilla, is a restriction on the size of pledgable wealth c (in their terminology, this represents the ability of banks to go after the borrowers' personal wealth in case of bankruptcy). Because, in our model, the principal screens out overoptimistic agents using high pay-for-performance sensitivity, our model suggests an alternative regulation approach of imposing a penalty

⁹See our footnote 7 on comparing our paper to theirs.

on high-powered incentives.¹⁰

In the equilibrium where the overoptimistic type is not hired, the presence of the overoptimistic agent still has an impact on social welfare in our model. Given the assumption that the principal's beliefs are correct, the presence of overconfident type increases both the wage expense of the principal and the utility of the competent agent by the same amount. The net impact on welfare depends on the individual welfare weights (there is no impact when the weights are equal). There is no counterpart of this result in Manove and Padilla because, in their model, there is no separating equilibrium where the overoptimistic type (agents with high signal) stays out of the market, but still impacts the contract with the competent type (agents with low signal).

Alternatively, one may evaluate each individual's utility using their own prior beliefs. Under this approach, all equilibria are Pareto-optimal because the agents enjoy their reservation utilities, and the principal chooses hiring and compensation to maximize the profit. The benefit of this approach is that it places value on efficient arbitraging of differences in beliefs.

Finally, in some cases, it may be reasonable to view the principal as overly pessimistic about the competent agent's ability. This may be appropriate when the agents are sophisticated business borrowers and the principal is a bank with little business-specific experience. Then, the presence of the overoptimistic type has no impact on social welfare if the type is not hired in equilibrium. If we also assume that not only type c but also type o has unbiased beliefs, then, for some parameter values, social welfare may be improved

¹⁰Manove and Padilla do not discuss regulation of incentives because, in their context, attention is naturally restricted to debt-type contracts)

by encouraging the principal to hire type o instead of type c .

4 Extensions

In our model of section 2, the principal has only one position, there are two types of candidates, and there are many candidates of each type. In this section, we show that the results are robust to introducing additional types, multiple positions, and letting the competent type c be pessimistic about own ability.

4.1 Adding Pessimistic and Incompetent Agents

Introducing additional type p agents who are pessimistic and incompetent, and have reasonably high reservation wage ($u_p \geq u_c$ is sufficient but not necessary), would not affect the equilibria described in Theorem 1. So long as the participation constraint for the new type of agent is above the two equilibria depicted in Figure 1, the new type of agent will not apply for any wage contract that appears in Equilibria VP and TS. Because the principal finds the new agent more expensive and less able than the other agents, the principal will not alter the wage contracts to attract the new agent. Adding other types of agents, however, may have some effect on the equilibrium. For example, adding a moderately confident agent with sufficiently low reservation utility will result in this agent accepting all wage contracts that can possibly attract anyone else (if the participation constraint line of the new agent is well below the equilibria depicted in Figure 1). If the principal views the new agent's ability as sufficiently low, the principal does not want to hire the agent, and mechanisms other than wage structure would be

required to screen out these agents.

4.2 Scarce Agents, Many Positions

When the principal has many positions and there is a shortage of agents, the wages that arise in equilibrium are similar to those highlighted in Figure 1, as we show next.

Suppose that at each time $t > 0$, the principal is approached by one agent. The principal believes that the agent is type o with probability π_o and type c with probability $1 - \pi_o$ (thus, knowing the types of past applicants does not inform the principal about the type of the current applicant). The principal offers the agent a menu of wage schedules, and the agent then decides whether to apply for one of them. Given two agent types, we can assume without loss of generality that the menu of wages consists of at most two wage schedules: one to attract type o and one to attract type c .

If the principal wishes to hire every agent, the principal's objective is to find the least expensive pair of wage schedules that would attract both types. The principal's problem is similar to the principal's problem in section 2, and thus leads to essentially the same wage schedules (we omit the formal derivation because it closely resembles that for Theorem 1). Specifically, depending on parameter values, the principal finds it optimal to either offer one wage schedule represented by point "CO" in Figure 1 to every agent, or offer a pair of wage schedules represented by points "Equilibrium VP" and "Equilibrium TS" in Figure 1, and let each agent choose which wage schedule to apply for. Hiring only one agent type is not an equilibrium in this setting because the principal could benefit by switching to hiring both types using one of the above wage menus (Assumption 1

guarantees feasibility of both wage menus). Specifically, hiring only type o (with wage schedule “VP”) is dominated by hiring both types with wage schedules “VP” and “TS,” and hiring only type c (with wage schedule “TS”) is dominated by hiring both types with wage schedule “CO”. Thus, there are two types of pure strategy symmetric equilibria:

Equilibrium CO At each time t , the principal offers one wage given by

$$(10) \quad \begin{aligned} w_h^* &= \frac{u_c + W(1 - q_c)}{q_c}, \\ w_l^* &= -W; \end{aligned}$$

the agent that arrives at time t applies and is hired with wage w^* .

Equilibrium TS_VP At each time t , the principal offers a menu of two wages, w_o^* and w_c^* , given by

$$(11) \quad \begin{aligned} w_{oh}^* &= \frac{u_o + W(1 - q_o)}{q_o}, \\ w_{ol}^* &= -W \end{aligned}$$

and

$$(12) \quad \begin{aligned} w_{ch}^* &= \frac{(1 - q_o)u_c - (1 - q_c)u_o}{q_c - q_o}, \\ w_{cl}^* &= \frac{q_c u_o - q_o u_c}{q_c - q_o}. \end{aligned}$$

If the agent that arrives at time t is type o , the agent applies for wage w_o^* and is hired; if the agent is type c , the agent applies for wage w_c^* and is hired.

In Equilibrium CO, the wage is represented by point “CO” in Figure 1. Thus, in this equilibrium, the agent of type c receives a reservation wage while the optimistic agent receives wage above the reservation level. Both agents are compensated only if the output is high, implying high performance sensitivity. In Equilibrium TS_VP, the equilibrium

wage is represented by points “Equilibrium VP” and “Equilibrium TS” in Figure 1. Thus, in this equilibrium, both agent types receive a reservation wage. The wage of agent c has low performance sensitivity: the agent is compensated both when output is high and when it is low. The wage of agent o , on the other hand, has high performance sensitivity: the agent is compensated only when the output is high. Note that hiring all agents allows the principal to let at least one type of agent bet their wealth on the outcome. This is an attractive bet to the principal, whose beliefs are more conservative than those of the agents.

As in the main model, which equilibrium is realized depends on the model parameters. Because in both equilibria the principal hires all agents, the principal chooses the equilibrium with the lowest expected wage. From the principal’s point of view, hiring type o is relatively cheaper, and hiring type c is relatively more expensive, in equilibrium TS_VP than in equilibrium CO. Thus, equilibrium TS_VP occurs when there are relatively many agents of type o (π_o is high), or when the differences in agents’ beliefs are large (q_o is a lot larger than q_c).

4.3 Pessimistic Competent Type

Our model assumes that both agent types are more optimistic about their own ability than the principal. The optimism of the competent type, however, is not crucial for our results. Specifically, if we alter the model by assuming that $q_c < f_c < q_o$ (instead of $f_c < q_c < q_o$), we obtain similar results, with the following alterations. Because the principal in this version is more optimistic than the competent type about the competent type’s ability, the optimal wage that attracts type c is flat: $w_l = w_h = u_c$. This implies

that, first, there is no pooling equilibrium, and second, the presence of the overconfident type does not impact the wage of the competent type when the competent type is hired in equilibrium.

There are several reasons, however, to view the original case as more interesting. First, the results in the original case are more robust to introduction of effort, where flat compensation becomes suboptimal. Second, the principal may have a preference for hiring optimistic agents. For example, Bolton, Brunnermeier, and Veldkamp (2009) argue that optimistic agents make better managers because they are better at coordinating actions and motivating their subordinates.

5 Conclusion

This paper looks at compensation mechanisms that attract talented managers when the principal disagrees with the agents' assessments of their own ability. In contrast with the common belief that stronger incentives attract more talented managers, we show that stronger incentives may instead attract overly confident managers. The principal may benefit from hiring the more confident instead of the more talented agent if the confident agent has deep pockets and is willing to bet the wealth as well as the future compensation on success.

Appendix

Proof of Theorem 1.

Proof. Consider a pooling equilibrium that attracts both types of agents.¹¹ The wage schedule in this equilibrium must be better for the principal than any other wage schedule. In particular, it must be better than any other wage schedule that attracts both types of agents. Therefore, the equilibrium wage schedule must solve the following problem:¹²

Problem 1.

$$\min_w (f_m w_h + (1 - f_m) w_l) \quad \text{subject to}$$

$$(13) \quad u_o \leq q_o w_h + (1 - q_o) w_l,$$

$$(14) \quad u_c \leq q_c w_h + (1 - q_c) w_l,$$

$$(15) \quad -W \leq w_l,$$

$$(16) \quad 0 \leq w_h - w_l,$$

where $f_m \equiv f_o \pi_o + f_c \pi_c$. In this problem, the constraints (13) and (14) insure that the best response of both types is to apply, as described in (2).

¹¹In this proof, we set aside the question of existence of such equilibrium. Existence is addressed in Theorem 2.

¹²While the objective in Problem 1 assumes that both agent types apply, some agents will be indifferent between applying and not applying if either (13) or (14) is satisfied with equality. Thus, we have to show that there is no equilibrium that does not solve Problem 1, in which the agents who are indifferent about the wage schedule solving Problem 1 do not behave as planned. The wage schedule in such an equilibrium would cost more to the principal than the wage schedule $w^* = (w_h^*, w_l^*)$ that solves Problem 1, and therefore would be dominated by the wage schedule $\hat{w} = (w_h^*, w_l^* + \varepsilon)$ with small enough $\varepsilon > 0$, which satisfies both (13) and (14) with strict inequality, and costs less to the principal for small enough $\varepsilon > 0$.

The solution $w^* = (w_l^*, w_h^*)$ to Problem 1 satisfies the following first order conditions with respect to w_h and w_l and complementary slackness conditions:

$$\begin{aligned}
f_m &= \lambda_1 q_o + \lambda_2 q_c + \lambda_4; \\
1 - f_m &= \lambda_1(1 - q_o) + \lambda_2(1 - q_c) + \lambda_3 - \lambda_4; \\
0 &= \lambda_1(q_o w_h + (1 - q_o)w_l - u_o); \\
0 &= \lambda_2(q_c w_h + (1 - q_c)w_l - u_c); \\
0 &= \lambda_3(W + w_l); \\
0 &= \lambda_4(w_h - w_l); \\
0 &\leq \lambda_k, \quad k=1,2,3,4.
\end{aligned}$$

Wage schedule w^* given by (8), combined with $\lambda_1 = \lambda_4 = 0$, $\lambda_2 = f_m/q_c$, and $\lambda_3 = 1 - f_m/q_c$, satisfies the above conditions. Therefore, w^* is the equilibrium wage schedule. Uniqueness of the solution follows from Lemma 1 (see the end of the Appendix).

Consider next a separating equilibrium. By definition, a separating equilibrium must attract either all agents of type o and none of type c or all agents of type c and none of type o . We analyze each of these two kinds of equilibrium separately, starting with the equilibrium that attracts only type o but not type c (the analysis of the other case is similar). The wage schedule in this equilibrium must be better for the principal than any other wage schedule. In particular, it must be better than any other wage schedule that attracts only agents of type o . Therefore, the equilibrium wage must solve the following problem:¹³

¹³While the objective in Problem 2 assumes that only type o applies, some agents will be indifferent between applying and not applying if either (17) or (18) is satisfied with equality.

Problem 2.

$$\min_w (f_o w_h + (1 - f_o) w_l) \quad \text{subject to}$$

$$(17) \quad u_o \leq q_o w_h + (1 - q_o) w_l,$$

$$(18) \quad u_c \geq q_c w_h + (1 - q_c) w_l,$$

$$(19) \quad -W \leq w_l,$$

$$(20) \quad 0 \leq w_h - w_l,$$

where the constraints (17) and (18) insure that the best response of type o is to apply and the best response of type c is to not apply, as described in (2).

The solution $w^* = (w_l^*, w_h^*)$ to Problem 2 satisfies the following first order conditions with respect to w_h and w_l and the complementary slackness conditions:

$$f_o = \lambda_1 q_o - \lambda_2 q_c + \lambda_4;$$

$$1 - f_o = \lambda_1 (1 - q_o) - \lambda_2 (1 - q_c) + \lambda_3 - \lambda_4;$$

$$0 = \lambda_1 (q_o w_h + (1 - q_o) w_l - u_o);$$

$$0 = \lambda_2 (u_c - q_c w_h - (1 - q_c) w_l);$$

$$0 = \lambda_3 (W + w_l);$$

$$0 = \lambda_4 (w_h - w_l);$$

$$\lambda_k \geq 0, \quad k=1,2,3,4.$$

Wage schedule (4) combined with $\lambda_2 = \lambda_4 = 0$, $\lambda_1 = f_o/q_o$, $\lambda_3 = 1 - f_o/q_o$ satisfies

This is not a problem, however, which can be seen following the logic of footnote 6.

the above conditions, and is therefore the equilibrium wage schedule. Uniqueness of the solution follows from Lemma 1 (see the end of the Appendix).

Consider next the equilibrium that attracts only type c but not type o . Similarly to the previous case, the wage in this equilibrium must solve the following problem:¹⁴

Problem 3.

$$\min_w (f_c w_h + (1 - f_c) w_l) \quad \text{subject to}$$

$$(21) \quad u_o \geq q_o w_h + (1 - q_o) w_l,$$

$$(22) \quad u_c \leq q_c w_h + (1 - q_c) w_l,$$

$$(23) \quad -W \leq w_l$$

$$(24) \quad 0 \leq w_h - w_l,$$

where the constraints (21) and (22) insure that the best response of type c is to apply and the best response of type o is to not apply (as described in (2)).

Similar to Problem 2, the solution $w^* = (w_l^*, w_h^*)$ to this problem satisfies the first order conditions with respect to w_h and w_l and four complementary slackness conditions corresponding to the four constraints. Wage schedule (6) satisfies these first order and

¹⁴While the objective in Problem 3 assumes that only type c applies, some agents will be indifferent between applying and not applying if either (21) or (22) is satisfied with equality. This is not a problem, however, as can be seen following the logic of footnote 6 and using $\hat{w} = (w_h - \varepsilon/q_c, w_l + \varepsilon/(1 - q_c) + (q_o - q_c)\varepsilon/q_c)$.

complementary slackness conditions and is therefore the equilibrium wage. As before, uniqueness follows from Lemma 1. \square

The following Lemma offers a simple sufficient condition for uniqueness of the solution to a linear program, which is used in the proof of Theorem 1.

Lemma 1. *Consider the following linear program:*

$$\min_{x \in R^n} a'x \quad \text{subject to}$$

$$b'_j x \geq c_j, \quad j = 1, \dots, J.$$

The solution x^ (which satisfies the first order condition $a = \sum_j \lambda_j b_j$ and the complementary slackness condition $\lambda_j(b'_j x^* - c_j) = 0$, $\lambda_j \geq 0$) is unique if (1) there are n indices j such that λ_j is not zero, and (2) the corresponding b_j 's are linearly independent.*

Proof. Suppose, on the contrary, there exists another solution $x \neq x^*$. By complementary slackness, $Bx^* = c$ where B is a matrix with rows b_j corresponding to the nonzero λ_j 's. By (1) and (2), B is invertible, implying that $Bx \neq c$ (otherwise, we would have $x = x^*$). Furthermore, $a'x - a'x^* = \sum \lambda_j b'_j x - \lambda_j b'_j x^* = \sum \lambda_j (b'_j x - c_j) > 0$ since by feasibility all terms are nonnegative and by $Bx \neq c$ some term must be strictly positive. Therefore x is dominated by x^* contradicting the optimality of x . \square

Table 1: Numerical Illustration of Possible Equilibria.

The numerical examples reported in this table illustrate that, depending on parameter values, any or all of the three sets of strategies (VP, TS, and NS, defined in Theorem 1) can form an equilibrium. Each line corresponds to a separate example. All of the examples also assume $q_c = 3/4$, $q_o = 1$, $W = 1$, $u_o = 7/6$, $u_c = 1$, $a_l = 10$, $a_h = 15\frac{1}{6}$, and each line specifies the remaining parameters π_c , f_o , and f_c and reports the principal's payoffs Π_o from hiring only type o , Π_c from hiring only type c , and Π_{ns} from hiring both types. Note that all of the assumed parameters satisfy assumptions 1a - 1d). As reported in the last column of the table, the resulting equilibria are those that correspond to the highest values of the principal's payoff.

Parameters			Principal's Payoff			Equilibria
π_c	f_o	f_c	Π_{vp}	Π_{ts}	Π_{ns}	
1/6	13/50	1/2	11.78	11.75	11.75	VP
2/15	1/4	5/8	11.75	12.3125	11.75	TS
1/2	1/4	1/2	11.75	11.75	11.9375	NS
1/10	1/4	1/2	11.75	11.75	11.6875	VP and TS
2/5	1/4	3/8	11.75	11.1875	11.75	VP and NS
7/15	1/8	1/2	11.375	11.75	11.75	TS and NS
1/5	1/4	1/2	11.75	11.75	11.75	VP, TS and NS

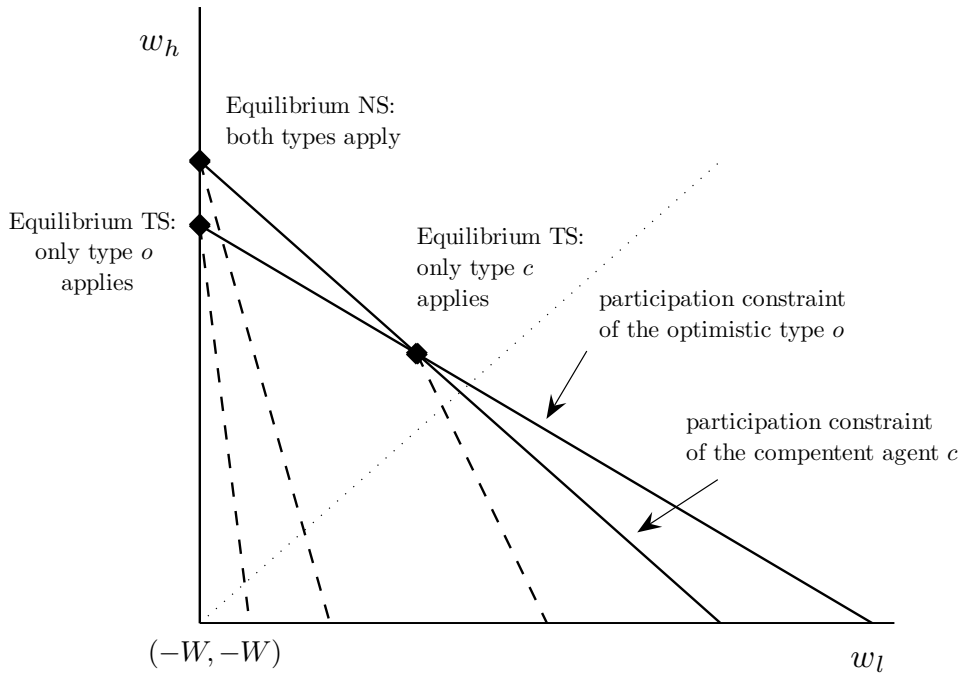


Figure 1: Model Analysis. The graph is built using the following parameter assumptions: $\pi_o = 0.9$, $u_o = 2.1$, $u_c = 2$, $q_o = 0.6$, $q_c = 0.5$, $f_o = 0.1$, $f_c = 0.3$, $W = 1$, $a_h - a_l = 10$. Given these parameters, there is only one separating equilibrium, Equilibrium VP (the optimistic agent is hired). If we instead assume $W = 0.5$, then there is a unique separating equilibrium, Equilibrium TS (the competent agent is hired). The corresponding figure is omitted since it is identical to the one shown, except that the axes move up and to the right so that they intersect at -0.5 rather than at -1 .

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