Financial Contracting and
Concentration of Operational Control*

Yu Wang
Boston University

Philip H. Dybvig
Washington University in Saint Louis

April 11, 2005

Abstract

Concentration of operational control can make it easier for a manager to operate more efficiently but can also make it easier for the manager to act in self interest counter to efficiency. This paper presents a stylized intertemporal model comparing two institutional arrangements, one with concentrated control and one with dispersed control. With concentrated control, the manager who chooses costly effort also controls the incoming cash flows, while with dispersed control the incoming cash flows are controlled by an investor, not the manager. The two arrangements create different potential incentive problems. With concentrated control, the manager may take the entire cash flow out of the firm and with dispersed control, the manager may expend too little effort. Depending on the relative severity of the two incentive problems, either arrangement may dominate. The incentive problems are exacerbated if there is an aftermarket in the firm’s assets and are reduced by gradual buyout of initial investment, compensation in ownership and non-vested pensions.

*This paper evolved from a draft circulated under the title of “Debt and Equity” that was also a chapter of Wang’s thesis, Wang (2001). We would like to thank Martin Cripps, Zsuzsanna Fluck, Mark Loewenstein, Kjell Nyborg, Jacob Oded, Anjolein Schmeits, and numerous seminar participants for valuable suggestions and comments. We are solely responsible for any errors in the paper. E-mail correspondence may be directed to Dybvig at pdybvig@dybfin.wustl.edu or Wang at wangyu1@bu.edu.
1 Introduction

The conflict between managers and outside financial claimants is a well-known source of economic inefficiency. This paper studies how the assignment of operational control rights can be used to minimize this conflict. The analysis focuses on two alternative institutional arrangements\(^1\) with different operational control rights in a stylized firm facing an infinite time horizon. In each arrangement, there is a manager who controls production and expends costly effort that affects the level of output. In one arrangement, there is \textit{concentrated control}, and the manager who controls production also controls revenues. In the other arrangement, there is \textit{dispersed control} and while the manager still controls production, an outside investor controls revenues. These two arrangements induce different incentive problems. With dispersed control, the manager may expend too little effort, while with concentrated control, the manager may appropriate this period’s cash flow.\(^2\) Depending on the relative severity of these two incentive problems, either arrangement may dominate the other. Allowing the manager to purchase a failed firm’s assets (an “aftermarket”) creates a new conflict of interest and tends to erode incentives in both cases; intuitively, the manager can “trash” the firm and buy it at a discount. Gradual buyout of initial investment, compensation in ownership, and non-vested pensions can all improve incentives in both arrangements.

When control is dispersed, the level of output is chosen by the manager. The output is collected by the investor, like cash at the door of a night club or a crop planted and cared for by a sharecropper but harvested by the owner. At the outset, the manager signs a wage contract that promises a fixed wage in each period. The investor is obligated to pay the fixed wage for as long as the manager is in charge of the project, but has full discretion to fire the manager at any time. Given that the manager’s income is guaranteed by the wage contract and does not depend on the firm’s output, the manager may have an incentive to expend no effort and just collect the wage. Depending on the parameter values, the manager’s shirking may or may not be deterred by the threat of dismissal.

When the manager is given the control over both the production and revenues, the manager is required to make a payment to the investor in each period. Failing to do so can lead to the dismissal of the manager. As the residual claimant on the firm’s current cash flow, the manager has no incentive to shirk, but may have an incentive to take the cash flow out of the firm and not to make the required payment. Depending

\(^1\)In earlier drafts, we interpreted these arrangements by their funding with debt or equity, but as suggested to us by Kjell Nyborg, it may be more instructive and illuminating to think of our analysis in terms of operational control rights.

\(^2\)In the stylized description we will use in the paper, this is outright theft, but of course in practice there are many more subtle forms of conflict of interest and we should not think of illegal activity as a necessary feature of the model.
on the parameter values, removal of the cash flow may or may not be deterred by the threat of dismissal.

The model implies that if rents from setting up the firm are very high, information problems are easy to overcome and either institution will work, while if rents are very low, neither institution is viable. What is more interesting is the case of intermediate profits in which only one of the two control arrangements is viable. In these cases, whether dispersed control or concentrated control is better depends on the relative severity of their incentive problems, which depends in turn on such determinants as the size of the initial investment, the future profitability of the project, how difficult effort is for the manager and the interest rate. With dispersed control, an increase in the manager’s difficulty of effort or a decrease in the project’s future profitability increases the manager’s benefit from shirking, and makes the manager’s incentive problem more severe. With concentrated control, an increase in the interest rate or a decrease in the project’s future profitability increases the manager’s benefit from taking the cash flow out of the firm, and therefore makes the manager’s incentive problem more severe.

An aftermarket in the firm’s assets creates a new conflict of interest and may jeopardize funding of the project, because investors may be concerned the manager will expend too little effort and then purchase the firm at a discount. In other words, firing the manager is less of a punishment with an aftermarket, because the manager can take over the firm and reinstate the project right after the dismissal. We use the term aftermarket as a shorthand for any sort of MBO, workout, or renegotiation that could return the manager to work at what amounts to the same firm.

Several institutions can be used to mitigate the incentive problems and support financing of the project. Dispersed control can be supported by compensation in ownership or managers’ non-vested pensions, while concentrated control can be supported by gradual buyout of initial investment. Compensating the manager in the firm’s ownership transfers the ownership from the investor to the manager and tends to reduce the manager’s incentive to shirk. Similarly, a non-vested pension can be held as collateral against shirking. Or, if a large enough fraction of the initial investment is bought out by the manager early enough, the payment to the remaining part of the investment can be guaranteed by the liquidation value of the project. To some extent, these results rely on an assumption that it is feasible to give each agent control over a fraction of output. In the sharecropping parable, this would be like having the manager and investor each harvest a fraction of the land. Or, in the nightclub parable,

---

3 As with all comparative statics results involving interest rates, we should be careful to think whether its application applies to real or nominal rates and whether the impact of the change would be confounded by a corresponding change in inflation or growth rates.

4 Although our model deals with the aftermarket for the firm’s assets, the effect is the same as the observation of Shapiro and Stiglitz (1984) regarding re-employment in the managerial labor market.
this would be like having the band and the club each collect the gate a fraction of the time or at a fraction of the entrances.

Our model predicts that dispersed control is more likely to be used by capital-intensive firms while concentrated control is more likely to be used by effort-intensive firms. The cash flow in a firm is the rental flow from the capital plus the fruits of the manager’s effort. In a capital-intensive firm, the relatively large rents to capital give a manager with concentrated control too much of a temptation to grab all the rents for one period even if that means giving up the future rents to management. This is why dispersed control is more likely to be used by capital-intensive firms. On the other hand, in a labor-intensive firm, dispersed control would give the manager too much temptation to shirk on effort against which firing might be an insufficient threat. In such a firm, this is a problem especially when the effectiveness of effort is low so that shirking on effort for one period gives more rents to the manager than the value of all subsequent wages.

Allocation of operational control rights is an important consideration in many practical contexts. In venture capital (and indeed in boutique investment banks offering mezzanine finance), the founding engineer/scientist is often given control of the “real” side, including production and R&D, while the venture capitalist has control of the “business” side including marketing, finance, and accounting. Therefore, a biotech consulting firm might have modest needs for capital (offices but no research equipment or manufacturing facilities). According to our model, the consulting firm would have concentrated control and get any (modest) necessary capital from a bank loan. On the other hand, a biotech manufacturing start-up with the same scientific talent but plans to set up research, development, and manufacturing in-house might require a large amount of capital. According to our model, the biotech manufacturing start-up would have dispersed control and would get the required (significant) necessary capital from a venture capitalist.

When interpreting the model, it is worth keeping in mind that we are assuming the extreme case in which the manager has no private funds at all to invest. For example, our model might suggest that the expected profit of a new restaurant established by a chef of unknown ability is small enough that our model would predict that neither institution would work. However, if the chef has a lot of own capital, using some may reduce the incentive to default on payments to an outside investor enough to make concentrated control viable. In this case the model would be interpreted as saying the restaurant is not viable unless the manager has personal funds to invest.

The recent literature has made it clear that the role of various contracts is much
different, and probably more realistic, in intertemporal settings than in the textbook single-period models. In our model, good managerial behavior is induced partially by a threat of termination or liquidation that will not be present in a single-period model. Examples of papers looking at corporate incentives in dynamic settings include Townsend (1979), Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990), Hart and Moore (1998), Fluck (1998), Myers (2000), DeMarzo and Fishman (2000), and Anderson and Nyborg (2001, 2002). One aspect of our analysis comes from an interesting idea of Fluck (1998): giving equityholders the right to fire the manager at will can be an effective way to align incentives to support outside equity. Myers (2000), which simplifies and extends Fluck (1998), is the basis for our model. In particular, the approach from Myers (2000) permits us to analyze incentives in a simple model without uncertainty.6

Our paper is related to some papers that focus on control rights within the firm.7 Zender (1991) looks at bankruptcy as a mechanism for transfer of control from equityholders to debtholders to avoid inefficient (dis)investment. Aghion and Bolton (1992) derive both conditions under which control is always given to one party and conditions under which there could be a transfer of control when there are potentially conflicting objectives between the manager and the investor. Kalay and Zender (1997) study the role of certain securities such as debt, equity, convertible debt, and warrant in allocating control to different parties in different contingencies to avoid suboptimal production decisions.

Our paper is also related to both papers that study venture capital financing (Gompers (1995), Hellman and Puri (2002) and Kaplan and Strömberg (2002)), which find that separated operational control is widely used in venture-capital-financed firms, and papers that study employee compensation, particularly the use of deferred compensation to affect employees’ decisions to expend effort and stay employed with the same firms (Lazear (1979), Friedberg and Owyang (2002), and Friedberg, Owyang, and Sinclair (2003) and many others).

The rest of the paper is organized as follows. Section 2 introduces the basic model in the paper (without an aftermarket) and includes subsections on dispersed control, concentrated control, and comparison of the two. Section 3 considers the model with an aftermarket and derive conditions under which the two controls are viable. Section 4 closes the paper.

---

6This means we cannot look at the asset substitution problem (i.e., the problem associated with the perverse incentive to substitute into a higher risk project to transfer value from bondholders to shareholders) that is often analyzed in single-period models, but we can still look at an interesting set of incentive issues in a tractable intertemporal model.

7Our paper does not appear to be closely related to the large literature on corporate control and the role of the board of directors.
2 The Basic Model

This paper studies how the assignment of operational control rights can be used to reduce potential economic inefficiency. To motivate the stylized model, we offer as a leading example a parable about the conflict between the manager of the farm and an outside investor.

We assume that the manager, who is also the entrepreneur who owns the project idea initially, is capable of operating a farm. The cash flows from the farm are as follows. Initial investment in the land occurs at time 0 and costs an amount \( K \). If the initial investment is made, the farm is in operation starting in year 1 through some liquidation date \( T_L \) (perhaps \( T_L = \infty \) indicating no sale) when the land is sold for \( K \). The manager has a unique talent for making the plants grow quickly by talking to the plants, but cannot communicate this knowledge. During each year \( t \) of operation, the farm generates a net cash flow (“income”)

\[
y_t = rK + \alpha e_t, \tag{1}
\]

where \( r > 0 \) is the riskfree rate, \( \alpha > 1 \) is a constant, and \( e_t \in [0, \bar{e}] \) is the manager’s effort (talking to the plants) at time \( t \). We interpret \( \alpha \) to be the manager’s efficiency. Absent effort, the farm earns a normal return \( rK \) and the net present value of the project is zero. Since \( \alpha > 1 \), the first-best has the manager undertaking the maximum effort \( \bar{e} \) in all periods. If the manager leaves the farm for a year, the speech that makes the plants grow fast is forgotten. In this case, effort afterwards will forever be 0 and the farm may as well be sold since it will never earn more than the riskfree rate. We assume that the manager earns a normalized zero wage working outside the farm and all the financial transactions between the manager and others can be verified at zero cost, but that contractual terms depending on nonpublic information rely on the agents’ voluntary compliance and cannot be enforced directly in court.

Since it is hard to solve information models even in a single-period context, we have chosen our assumptions to simplify the problem. One simplifying assumption is that there is no uncertainty in the model and the information problem is modeled using actions that are “observable but not contractible.” This assumption has been around for a long time, and was used in the model of Myers (2000) we are following closely. Nobody believes it is a literal description of reality that there is no learning over time or inference from actions, but this simplification allows us to make the model richer in other respects. Linearity along rays of the technology and preferences is another useful simplification. This implies that if the institution can induce effort at any scale then it can induce first-best effort and operate at full scale. We do not take

---

8See, for example, Dybvig and Spatt (1981).
the first-best result literally, since nobody believes anything in the economy really achieves the first-best. Nonetheless, the assumption of linearity is useful because it simplifies the analysis and gives easily interpretable somewhat stark directional results.

The manager maximizes the total present value of consumption less effort, with discounting at the riskfree rate \( r \). For the manager, feasible consumption \( c_t \) at each \( t \) is nonnegative and feasible effort \( e_t \) is constrained to the interval \([0, \bar{e}]\). The manager has no endowment and therefore must turn to an investor to finance the project. The investor maximizes the net present value of consumption and is truly risk-neutral (unlike the manager who is risk-neutral for nonnegative consumption and in effect infinitely risk averse for negative consumption). We assume the manager has the unique skill at the outset and captures all rents.

**Dispersed Control**

With dispersed control, the manager chooses costly effort and the outside investor collects the revenues. At the outset (time 0), the investor pays \( K \) to buy the parcel of land and hires the manager to operate the farm. The investor controls the revenues, and the manager receives a fixed wage in each period working for the firm as enforced by a wage contract. Later we allow an aftermarket in the firm’s assets, but not in this section.

The game between the manager and the investor is a game of perfect information with sequential moves. In each period \( t = 1, 2, ..., T_L \), we have the following stages:

1. The manager chooses effort \( e_t \in [0, \bar{e}] \).
2. Output \( rK + \alpha e_t \) is realized.
3. The investor collects the output and pays the manager the pre-agreed wage \( w_t \geq 0 \) which is consumed by the manager: \( c_t = w_t \).
4. The investor chooses whether to liquidate: the indicator \( l_t = 1 \) if the investor chooses to liquidate the firm at time \( t \), while \( l_t = 0 \) indicates no liquidation.
5. If the investor chooses \( l_t = 1 \), then \( T_L = t \) and the game ends with the investor collecting \( K \) from sale of the assets. Otherwise, the game continues in period \( t+1 \).

At each stage, the decision can depend on the entire history known to the agent. We use capital letters to denote histories: \( E_t \equiv (e_1, ..., e_t) \) and \( L_t \equiv (l_1, ..., l_t) \). The
manager’s choice of effort at time $t$ depends on $E_{t-1}$. It might seem that the history should depend on $L_{t-1}$ as well, but if $l_s \neq 0$ for any $s < t$, the game would be over. Therefore, if there is a choice today it must be that $L_{t-1} = 0$ and this does not have to be included as an argument to the manager’s strategy. Similarly, the investor’s choice on whether to liquidate at $t$ depends on $E_t$ but we do not have to include $L_t$ as an argument. The investor’s choice at $t$ depends on $E_t$ rather than $E_{t-1}$ because the investor learns the manager’s choice of effort earlier in the period before it is time to choose whether to liquidate.

Strategies are denoted by $\sigma$’s: $\sigma^e_t(E_{t-1})$ is the function giving the manager’s choice of effort $e_t$ at time $t$ and $\sigma^l_t(E_t)$ is the function giving the investor’s choice of whether to liquidate $l_t$ at time $t$. Here are the formal choice problems:

**Problem 1 Manager’s problem (dispersed control without an aftermarket)** Given the contract terms $\{w_t\}_{t=1}^{\infty}$ and the investor’s strategy $\{\sigma^l_t\}_{t=1}^{\infty}$ for liquidation, choose a strategy $\sigma^e_t : \mathbb{R}^{t-1} \rightarrow [0, \infty]$ for effort for each $t = 1, 2, \ldots, T_L \equiv \min\{t | l_t = 1\}$, to maximize

$$\sum_{t=1}^{T_L} \frac{c_t - e_t}{(1 + r)^t}$$

subject to, for all $t = 1, 2, \ldots, T_L$,

$$e_t = \sigma^e_t(E_{t-1}),$$

$$c_t = w_t,$$

$$l_t = \sigma^l_t(E_t).$$

**Problem 2 Investor’s problem (dispersed control without an aftermarket)** Given the contract terms $\{w_t\}_{t=1}^{\infty}$ and the manager’s strategy $\{\sigma^e_t\}_{t=1}^{\infty}$, choose strategies $\sigma^l_t : \mathbb{R}^t \rightarrow \{0, 1\}$ for liquidation to maximize

$$\sum_{t=1}^{T_L} \frac{rK + \alpha e_t - w_t}{(1 + r)^t} + \frac{K}{(1 + r)^{T_L}} - K$$

subject to, for all $t = 1, 2, \ldots, T_L$,

$$e_t = \sigma^e_t(E_{t-1}),$$

$$c_t = w_t,$$

$$l_t = \sigma^l_t(E_t).$$

The equilibrium concept is subgame perfect Nash equilibrium, so there will be a similar problem to be solved at each date and for every possible history, and not just
for the histories that would be realized along the equilibrium path. This standard equilibrium concept seems like a reasonable one in this model (what further justification do we have for different equilibrium concepts?) and puts reasonable discipline on expectations about how the other player will continue if one player deviates from the candidate optimal strategy.

Because the marginal benefit ($\alpha$) of effort is larger than the marginal cost ($1$), the first-best has maximal effort $e_t = \tau$ at all times $t$. Due to the stark linearity of the model, we will have equilibria that are either completely inefficient (with no surplus available from formation of a firm) or completely efficient.

The formal game does not include the contracting stage before the firm is formed. We think of the manager as having the unique talent and therefore all the bargaining power in this stage. Reflecting this view, the contracts we are interested in have maximum surplus for the manager and zero surplus for the investor.

Here is our main result characterizing the equilibrium in this case.

**Theorem 1 Equilibrium (dispersed control without an aftermarket)** In the basic model with dispersed control, the equilibrium depends on the profitability of the project. If the profitability is no less than the single-period cost of maximal effort, a contract can be chosen that implements the first-best, while otherwise no contract offers positive surplus. Specifically:

- If $(\alpha - 1)\tau/r \geq \tau$, the first-best effort is achieved in equilibrium given the contract specifying wages $w_t = \tau \alpha$ for all $t$ and strategies

$$\sigma^s_t(E_{t-1}) = \begin{cases} \tau & \text{if } e_s = \tau \text{ for all } s < t \\ 0 & \text{otherwise} \end{cases}$$

and

$$\sigma^l_t(E_t) = \begin{cases} 0 & \text{if } e_s = \tau \text{ for all } s < t \\ 1 & \text{otherwise} \end{cases}$$

This contract and equilibrium give zero surplus for the investor and maximal surplus $\tau (\alpha - 1)/r$ to the manager.

- If $(\alpha - 1)\tau/r < \tau$, then every equilibrium with nonnegative surplus for both agents has zero effort in all periods and zero surplus for both agents.

**Proof:** The formal proof is given in the appendix. Here is some of the intuition. If $\alpha \geq 1 + r$ and the posited contract is in place, then the manager’s loss from

\[9\text{The choice problem starting from a node takes as given an arbitrary history and then specifies dynamics from that point on using the strategy functions.}\]
deviation, which is the present value of future salary \( \bar{v}(\alpha - 1)/r \), is greater than the savings in effort this period, which is \( \bar{v} \). If the condition is violated and \( \alpha < 1 + r \), then for any candidate contract with positive effort there has to be a period when it would pay for the manager or the investor to deviate. Intuitively, promises of higher salary would have to be so large in this case that sooner or later either the promises would grow too large for the investor to want to continue or the investor will find insufficient inducement to keep making effort.

The agents’ strategies in the good equilibrium with dispersed control consist of two parts: the good strategies (played on the equilibrium path) and the bad or punishment strategies (played on every off-equilibrium path). The first-best effort on the equilibrium path is supported by the threat to play the punishment strategies forever once one party deviates from the good strategy, and can be supported only when \( \alpha \geq 1 + r \), i.e., when the effectiveness of effort is large enough. As is normal in subgame perfect Nash Equilibrium, the off-equilibrium strategy of an agent is important because it describes expectations about what would happen after a deviation. Assuming no effort after a deviation makes credible the extreme punishment that might be needed to support equilibrium. Having this equilibrium puts discipline on the off-equilibrium paths, unlike Nash equilibrium in which there is no restriction on strategies off the equilibrium path.

**Concentrated Control**

With concentrated control, the manager both chooses the costly effort and collects the revenues. At the outset (time 0), the investor pays \( K \) to buy the parcel of land and arranges for the manager to operate the farm. The manager is committed to make a payment to the investor in each subsequent period. The investor has full discretion to liquidate the firm to recover the investment \( K \) any time the manager fails to make the promised payment. In the model in this section, there is no aftermarket.

The game between the manager and the investor is a game of perfect information with sequential moves. In each period \( t \geq 1 \), we have the following stages:

1. The manager chooses effort \( e_t \in [0, \bar{e}] \).
2. Output \( rK + \alpha e_t \) is realized.
3. The manager collects the entire output, and then chooses the payment \( p_t \) to the investor and consumes the remainder: \( c_t = rK + \alpha e_t - p_t \).

---

10 With concentrated control, the investor is like a debtholder and the only way to penalize the manager for not making the promised payment(s) is to force liquidation of the firm.
4. The investor decides whether to liquidate: the indicator $l_t = 1$ if the investor chooses to liquidate, while $l_t = 0$ indicates no liquidation. If the payment meets or exceeds the promise ($p_t \geq \pi_t$) this period, then the investor cannot liquidate and must choose $l_t = 0$.

5. If the investor chooses $l_t = 1$, then $T_L = t$ and the game ends with the investor collecting $K$ from sale of the assets. Otherwise, the game continues in period $t + 1$.

At each stage, the decision can depend on the entire history known to the agent. As before, we use capital letters to denote histories: $E_t \equiv (e_1, ..., e_t)$, $P_t \equiv (p_1, ..., p_t)$ and $L_t \equiv (l_1, ..., l_t)$. The manager’s choices of effort and payout at time $t$ depend on $P_{t-1}$ and $E_{t-1}$ (and again it need not depend on $L_{t-1}$ since if there were $l_s \neq 0$ for any $s < t$ then the game would be over). Neither effort at $t$ nor payout at $t$ depends on the other because these are simultaneous moves. The investor’s choice of whether to liquidate depends on $P_t$ and $E_t$. These have subscripts $t$ rather than $t - 1$ because the manager knows the effort and payment at $t$ before choosing whether to liquidate at $t$. Note that the payment $P_{t-1}$ is assumed to be contractible (e.g. notarized or a payment by check), which is why it is only feasible for the investor to liquidate in event of a default. This is like a bond contract.

Unlike the investor’s absolute commitment to pay wages to the manager in the case of dispersed control, the manager cannot make an absolute commitment to make payments to the investor. We are thinking that the investor has “deep pockets” and has collateral or other resources that can be attached to guarantee a wage payment, while the manager has limited financial resources. This asymmetry is why the manager cannot finance the firm directly and must go to the investor to raise money. It also means that if the manager chooses not to make a payment, the only recourse is to terminate the firm.

Strategies are denoted by $\sigma$’s: $\sigma^e_t(E_{t-1}, P_{t-1})$ is the function giving the manager’s choice of effort at time $t$, $\sigma^p_t(E_{t-1}, P_{t-1})$ is the function giving the manager’s choice of payment as a fraction of available income at time $t$, and $\sigma^l_t(E_t, P_t)$ is the function giving the investor’s choice of whether to liquidate at time $t$.

**Problem 3 Manager’s problem (concentrated control without an aftermarket)**

Given the contract terms $\{\pi_t\}_{t=1}^{\infty}$ and the investor’s strategy $\{\sigma^l_t\}_{t=1}^{\infty}$ for liquidation, choose strategies $\sigma^e_t : \mathbb{R}^{t-1} \times \mathbb{R}^{t-1} \rightarrow [0, \bar{c}]$ for effort and $\sigma^p_t : \mathbb{R}^{t-1} \times \mathbb{R}^{t-1} \rightarrow [0, rK + \alpha e_t]$ for payment for each $t = 1, 2, ..., T_L \equiv \min\{t|l_t = 1\}$, to maximize

$$\sum_{t=1}^{T_L} \frac{c_t - e_t}{(1 + r)^t}$$

subject to, for all $t = 1, 2, ..., T_L$.
\[ e_t = \sigma^e_t(E_{t-1}, P_{t-1}), \]
\[ p_t = \sigma^p_t(E_{t-1}, P_{t-1}), \]
\[ c_t = rK + \alpha e_t - p_t, \text{ and} \]
\[ l_t = \sigma^l_t(E_t, P_t). \]

**Problem 4 Investor’s problem (concentrated control without an aftermarket)**

Given the contract terms \( \{\pi_t\}_{t=1}^\infty \) and the manager’s strategy \( \{\sigma^e_t\}_{t=1}^\infty \) and \( \{\sigma^p_t\}_{t=1}^\infty \), choose strategies \( \sigma^l_t : \mathbb{R}^t \times \mathbb{R}^t \rightarrow \{0, i(p_t < \pi_t)\} \) for liquidation to maximize
\[
\sum_{t=1}^{T_L} \frac{p_t}{(1 + r)^t} + \frac{K}{(1 + r)^{T_L}} - K
\]
subject to, for all \( t = 1, 2, ..., T_L \),
\[ e_t = \sigma^e_t(E_{t-1}, P_{t-1}), \]
\[ p_t = \sigma^p_t(E_{t-1}, P_{t-1}), \]
\[ c_t = rK + \alpha e_t - p_t, \text{ and} \]
\[ l_t = \sigma^l_t(E_t, P_t). \]

Here is our main result characterizing the equilibrium in this case.

**Theorem 2 Equilibrium (concentrated control without an aftermarket)** *In the basic model with concentrated control, the equilibrium depends on the profitability of the project. If the profitability is no less than the single-period rent to capital, a contract can be chosen that implements the first-best, while otherwise no contract offers positive surplus. Specifically:

- If \((\alpha - 1)\bar{c}/r \geq rK\), the first-best effort is achieved in equilibrium given the contract specifying payments \( \pi_t = rK \) for all \( t \) and strategies
  \[
  \sigma^e_t(E_{t-1}, P_{t-1}) = \bar{c} \quad \text{for all } t, \]
  \[
  \sigma^p_t(E_{t-1}, P_{t-1}) = \begin{cases} 
  rK & \text{if } p_s = rK \text{ for all } s < t \\
  0 & \text{otherwise}
  \end{cases}
  \]
  and
  \[
  \sigma^l_t(E_t, P_t) = \begin{cases} 
  0 & \text{if } p_s = rK \text{ for all } s \leq t \\
  1 & \text{otherwise}
  \end{cases}
  \]
- If \((\alpha - 1)\bar{c}/r < rK\), then every equilibrium with nonnegative surplus for both agents has zero payments along the equilibrium path and zero surplus for both agents.*
PROOF: The formal proof is given in the appendix. Here is some of the intuition. The manager always wants to choose $e_t = \bar{e}$ along the equilibrium path, since following any other strategy is dominated by increasing $e_t$ to $\bar{e}$ and consuming the additional output while keeping the payments unchanged. If $(\alpha - 1)\bar{e}/r \geq rK$, then the manager wants to make the full payment $rK$ to keep the firm in operation. The expected payoff from doing so, which is equal to the present value of future profit of the firm $(\alpha - 1)\bar{e}/r$, is larger than that from deviation, which is $rK$. If $(\alpha - 1)\bar{e}/r < rK$, then the expected payoff from deviation is larger than that from making the full payment. As a result, no payment would be made along the equilibrium path.

Dispersed Control versus Concentrated Control

Our analysis in the previous two subsections shows that the two operational control arrangements considered in the paper induce different incentive problems. When control is dispersed, the manager controls the production and the investor controls the revenues. Given that the manager’s income in a period is guaranteed by the wage contract and does not depend on the firm’s output, the manager may have an incentive not to exert any effort on the project and just collect the wage. Depending on the parameter values, the manager’s shirking may or may not be deterred by the threat of dismissal.

When control is concentrated, the manager controls both production and revenues, and is required to make a payment to the investor in every period. As the residual claimant on the firm’s current cash flow, the manager has no incentive to shirk, but may have an incentive to take the entire cash flow out of the firm and not to make any payment to the investor. Depending on the parameter values, removal of the cash flow may or may not be deterred by the threat of dismissal.

The analysis in the previous two subsections produces the following result on the viability and optimality of the two controls:

**Theorem 3** Dispersed control versus concentrated control We have that (i) neither dispersed control nor concentrated control can be used when $(\alpha - 1)\bar{e}/r < \min\{rK, \bar{e}\}$; (ii) concentrated control, but not dispersed control, can be used when $rK \leq (\alpha - 1)\bar{e}/r < \bar{e}$; (iii) dispersed control, but not concentrated control, can be used when $\bar{e} \leq (\alpha - 1)\bar{e}/r < rK$; and (iv) both controls can be used when $\max\{rK, \bar{e}\} \leq (\alpha - 1)\bar{e}/r$.

PROOF: Immediate from Theorem 1 and Theorem 2.
In the statement, “can be used” means the institution can implement the first-best or equivalently any positive effort (see Theorem 1 and Theorem 2 for more precision). The term \((\alpha - 1)\bar{c}/r\) equals the value of the firm to the manager. It also equals the loss that the manager incurs when leaving the firm. The terms \(\bar{c}\) and \(rK\) equal respectively the gain for the manager from shirking and the gain from not making the promised payment. The theorem shows that dispersed control (concentrated control) can be used only when the manager cannot benefit from shirking (not making the promised payment) and that depending on the relative severity of the two incentive problems, either control can dominate the other.

The model has the following predictions on the optimal choice between the two controls:

- When potential economic profits are small, incentive problems prevent either institution from being viable.
- When potential economic profits are large, either institution provides good incentives.
- In the more interesting case of intermediate economic profits, one or the other institution can be better:
  1. Dispersed control is preferred when production is capital-intensive.
  2. Concentrated control is preferred when production is effort-intensive.

In the farming parable, these predictions imply that dispersed control, not concentrated control, should be used when it is not a lot of work for the manager to talk to the plants, but the initial investment on the farm is large relative to the future profit of the farm. Concentrated control instead of dispersed control, on the other hand, should be used when it is a lot of work to talk to the plants, but the initial investment of the farm is small relative to the future profit of the farm.

It is worth keeping in mind that the model assumes the manager has no capital. If the manager has some capital, this in effect reduces the size of invested needed in the firm and makes concentrated control easier to implement. If the manager has enough capital to fund the entire project, then this internalizes the externalities and the concentrated control can always be used.

Contracts that offer the same operational control rights as the ones considered in the paper are often used for various projects. Most debt contracts, for example, give the control over both the production and revenues to managers. Many contracts used by venture capitalists, on the other hand, give the control over the production (i.e. daily operation of the firms) to managers and the control over (at least part of) the
cash flows to investors (Gompers (1995), Kaplan and Strömberg (2002) and Hellman and Puri (2002)). One reason why dispersed control is used in many venture-capital-financed firms is that managers in these firms often have a large discretion over the firms’ spending and that giving the control over the cash flows to investors would restrict the managers’ ability to use the firms’ resources to benefit themselves.

3 Impact Of An Aftermarket

The rest of the paper extends the basic model to include terminal conditions. We look at particularly what happens to incentives when there is an after-market in the firm’s assets that might permit the manager to restart the firm. Like the possibility of a workout or an MBO, an aftermarket can undermine the incentives by removing the loss when the firm fails. This phenomenon is similar to the observation for labor markets of Shapiro and Stiglitz (1984), although it operates through the capital market. The incentives can be restored by institutions, such as gradual buyout of initial investment, compensation in ownership, and managers’ non-vested pensions. First we consider dispersed control. Then we consider concentrated control.

Dispersed Control with an Aftermarket

In the example of a farm considered in the previous section, we have the following trade-off when the manager has worked long enough to accumulate the funds needed to buy the firm (assuming that neither compensation in ownership nor the manager’s non-vested pension is used). The gain from shirking for the manager equals the costly effort $e$. The loss from shirking equals the part (say $f$) of the future value of the project that cannot be recaptured by buying up the firm at market $f(\alpha - 1)e/r$. $f$ is often determined by the parties’ relative bargaining power after a failure and can take any value between zero and one: perhaps $f \approx 0$ if there are many close substitutes that can be purchased, and perhaps $f \approx 1$ if the outside investor owns many such projects, as in the chain-store paradox (Selten (1978), Milgrom and Roberts (1982), Kreps and Wilson (1982), and Fudenberg, Kreps, and Maskin (1990)). Dispersed control can be used if and only if $f(\alpha - 1)e/r \geq e$. When $f < 1$, dispersed control works less well in this case than without the aftermarket. Furthermore, if the capital in the firm can be purchased at its value in its best alternative use $K$, then $f = 0$ and dispersed control cannot be used.

Managers’ non-vested pensions can be used to reduce the severity of the incentive problem and support financing of the project. The basic idea behind this result is quite
simple: One way to prevent the manager from benefiting from a deviation from the first-best effort is to create an offsetting loss, which can be the loss of a pension that is awarded in full only if the manager stays with the firm until retirement.\textsuperscript{11}

Compensation in ownership can also be used to support dispersed control, provided it is possible to split the firm’s output between the manager and the investor. In the case of a farm, the manager and the investor might be given a fraction of the acreage to harvest (chosen randomly if there is an issue of differential effort). In the case of a club, management could collect the gate at a fraction of the entrances or a fraction of the time, with the band collecting the rest of the time. Absent compensation in ownership, the manager accumulates outside assets over time, and these assets can undermine incentives because they make it possible for the manager to buy the firm’s assets in the aftermarket. However, compensation in ownership means that by the time the manager is rich enough to purchase the firm, the manager already owns it. If you are the owner of a firm, cheating the firm is only cheating yourself!

When compensation in ownership is used, the manager and investor play a game with an infinite number of periods, or more precisely an infinite number of periods if the firm is not liquidated in finitely many periods. The game played also depends on the contract that is in place, which specifies the cash payment (possibly in the form of output) and share grant the investor is committed to give the manager in each period the manager works (whatever the effort level). The promised cash payment at time $t$ is $\kappa_t$ and the cumulative share grant, as a fraction of the firm, is $\beta_t$. The (cumulative) share grant at time $t - 1$ provides a fraction of output $\beta_{t-1} (rK + \alpha e_t)$ in period $t$, part or all of which could be given to the manager in shares as part of the share grant given to the manager in period $t$. The share grant in period $T_L - 1$ also provides part or all the liquidation value $\beta_{T_L} K$ at time $T_L$ (if $T_L < \infty$) when the firm is liquidated. At the start of the game, it is assumed the contract is in place and the initial investment $K$ in capital has already been made.

In each period until liquidation of the firm, first the manager chooses effort $e_t$ and consumption $c_t$. As payment, the manager receives cash $\kappa_t$ and additional shares $\beta_t - \beta_{t-1}$, both commitments made in the contract and binding as long as the firm operates. The manager’s cash $m_{t-1}$ from the previous period also pays interest $rm_{t-1}$. The new cash balance (not permitted to go negative) is the old balance plus interest plus cash payment, less consumption. The last thing that happens in the period is the investor’s

\textsuperscript{11}In the US, ERISA restricts the length of the period over which the pension is not vested. However, this restricts legal vesting, not the more important notion of economic vesting. As noted by many authors (e.g. Lazear (1979)), the obligation in traditional defined-benefit plans accumulates slowly in early years and penalizes workers who change jobs. In other words, there is less than full economic vesting even if the pension is fully vested in a legal sense. There is an extensive literature on pensions and incentives in the labor literature. Lazear (1979), Friedberg and Owyang (2002), and Friedberg, Owyang, and Sinclair (2003) are just a few examples of papers showing how non-vested pensions can affect incentives within the firm.
decision to liquidate the firm, which is the only recourse if effort is substandard. If the firm is liquidated, the manager receives the share $\beta_t K$ and the investor receives the remainder $(1 - \beta_t) K$. If the manager has enough money ($K$) after consumption to buy the firm’s assets, the manager buys the firm’s assets in the aftermarket, running it without any incentive problem since it is solely owned without debt, for a net surplus having present value $\alpha(\bar{e} - 1)/r$ at the liquidation date. If the manager does not have enough funds to buy the assets in the aftermarket, we assume the manager is not trusted and cannot raise new funds to continue operations.

This paper considers a contract that pays both wage and dividend in shares except at the time of liquidation. (It can easily be shown that some other contracts can also be used to support dispersed control.) This is the contract that allows the ownership to be transferred from the investor to the manager at the fastest possible rate. The cash payments and cumulative share grant in this contract are given by

$$\kappa_t = \begin{cases} 0 & \text{if } l_t = 0 \\ \beta_{t-1}(rK + \alpha e_t) & \text{otherwise} \end{cases}$$

$$\beta_0 = 0$$

$$\beta_t = \beta_{t-1} + \Delta \beta_t$$

where

$$\Delta \beta_t = \begin{cases} \alpha \bar{e}/K + r \beta_{t-1} & \text{if } l_t = 0 \\ (1 - \beta_{t-1})\alpha \bar{e}/K & \text{otherwise} \end{cases}$$

is the shares given to the manager in period $t$.

The intuition behind this contract is as follows. When the manager exerts the maximal effort, the first-best outcome is achieved. In this circumstance, the best way to compensate the manager is to give the manager both the wage $\alpha \bar{e}$ and dividend $\beta_{t-1} r K$ in shares and let the investor to collect all the output. When a less-than-the-maximal-effort effort is exerted, liquidation would occur in equilibrium. An easy way to divide the output between the two parties is to ask each to collect the part that belongs to that party. In this circumstance, only part of the wage $(1 - \beta_{t-1})\alpha \bar{e}$ is given in shares. Both the remaining part of the wage and dividend are given in cash in the form of output. \(^{13}\)

\(^{12}\)It is possible that with this contract, the manager’s ownership of the firm may exceed 100% (i.e. $\beta > 1$) at some point over time. If that occurs, it means that the transfer of ownership is complete and the investor needs to pay the manager the part that is above 100%.

\(^{13}\)It can easily be shown that the contract in (2) can also be used for a night club, even though
The players’ strategies are described by strategy functions $\sigma^e_t$ (for effort), $\sigma^c_t$ (for consumption as a fraction of available cash), and $\sigma^l_t$ (as liquidation indicator: 1 indicates liquidation and 0 indicates no liquidation). The solution concept for the game is subgame perfect Nash equilibrium. This is a game of perfect information, since each player knows the entire history of moves at all times. As a result, all subgames are proper, and subgame perfection says that the continuation is a Nash equilibrium given any hypothetical history.

Here is the manager’s choice problem at the start of the game. It is easy to write down the similar choice problem that corresponds to the subgame starting at any choice node.

**Problem 5 Manager’s problem (dispersed control with an aftermarket):** Given the contract terms $\{\beta_t\}_{t=0}^\infty$ and $\{\kappa_t\}_{t=1}^\infty$ and the investor’s strategy $\{\sigma^l_t\}_{t=1}^\infty$ for liquidation, choose strategies $\sigma^e_t : \mathbb{R}^{2(t-1)} \rightarrow [0, \bar{c}]$ for effort and $\sigma^c_t : \mathbb{R}^{2(t-1)} \rightarrow [0, 1]$ for consumption for each $t = 1, 2, ..., T_L$, to maximize

$$
\sum_{t=1}^{T_L} \frac{c_t - e_t}{(1 + r)^t} + \frac{1}{(1 + r)^{T_L}} \left( \frac{(\alpha - 1)\bar{c}K}{r} + (\beta T_L K + m T_L) \right)
$$

subject to:

- $m_0 = 0$,
- and for all $t = 1, 2, ..., T_L$:
  - $e_t = \sigma^e_t(E_{t-1}, C_{t-1})$,
  - $c_t = (m_{t-1}(1 + r) + \kappa_t)\sigma^c_t(E_{t-1}, C_{t-1})$,
  - $l_t = \sigma^l_t(E_t)$,
  - $m_t = m_{t-1}(1 + r) + \kappa_t - c_t \geq 0$, and
  - $T_L = \min\{t|l_t = 1\}$.

the manager of the night club does receive cash payment (by collecting the gate at the entrances of the club) in each period. This can be done by requesting the manager to purchase the shares of the firm with all the cash the manager receives in each period and threatening to fire the manager if the manager refuses to do so.

14Relaxing this assumption (i.e. allowing the manager to consume the wealth in shares) will not change the results of the paper, because (in every equilibrium in which shirking never occurs) the manager is indifferent between consuming the share wealth before and after taking over the firm is completed.

15It seems reasonable to assume that the manager’s consumption cannot be used to make the liquidation decision by the manager (maybe it is because the investor cannot observe the manager’s consumption). Otherwise, the first-best can easily be achieved by basing the liquidation decision on the manager’s consumption, for example, by choosing to liquidate the project if the manager does not consume all the income in each period.
In the choice problem, we use capital letters to indicate histories of consumption and effort: \( E_t \equiv (e_1, e_2, \ldots, e_t) \) and \( C_t \equiv (c_1, c_2, \ldots, c_t) \). Also, \( \iota(\cdot) \) is an indicator function:

\[
\iota(\beta_{TL} K + m_{TL} \geq K) = \begin{cases} 
1 & \text{if } \beta_{TL} K + m_{TL} \geq K \\
0 & \text{otherwise}
\end{cases}
\]

**Problem 6 Investor’s problem (dispersed control with an aftermarket):** Given the contract terms \( \{\beta_t\}^T_L \) and \( \{\kappa_t\}^T_L \) and the manager’s strategy \( \{\sigma^c_t\}^T_L \) for effort, choose a strategy \( \sigma^l_t : \mathbb{R}^T \rightarrow \{0, 1\} \) for liquidation for each \( t = 1, 2, \ldots \), to maximize

\[
\frac{\sum_{t=1}^{T_L-1} rK + \alpha e_t}{(1 + r)^t} + \frac{(1 - \beta_{TL-1})(rK + \alpha e_{TL}) + (1 - \beta_{TL})K}{(1 + r)^{TL}} - K
\]

subject to:

\[
m_0 = 0, \\
\text{and for all } t = 1, 2, \ldots, T_L: \\
e_t = \sigma^c_t(E_{t-1}, C_{t-1}), \\
c_t = (m_{t-1}(1 + r) + \kappa_t)\sigma^c_t(E_{t-1}, C_{t-1}), \\
l_t = \sigma^l_t(E_t), \\
m_t = m_{t-1}(1 + r) + \kappa_t - c_t \geq 0, \text{ and} \\
T_L = \min\{t | l_t = 1\}.
\]

Here is our main result characterizing the equilibrium in this case.

**Theorem 4 Equilibrium (dispersed control with aftermarket)** In the model of concentrated control with an aftermarket, whether the first-best outcomes can be achieved depends both on how large the effectiveness of effort \( \alpha \) is and on whether compensation in ownership or manager’s non-vested pension is used. Specifically:

- **Dispersed control cannot be supported by the threat of dismissal itself:** When neither compensation in ownership nor manager’s non-vested pension is used, the first-best effort cannot be achieved, and in fact no equilibrium has positive effort along the equilibrium path.

- **Dispersed control can be supported by compensation in ownership:** When the contract in (2) is used and when

\[
1 + r \leq \alpha < \frac{K}{\epsilon}
\]
and

\[
\beta_{T_{mbo}}^{-1} = \frac{\alpha \bar{e}(1 + r)^{T_{mbo}^{-1}} - 1}{rK} \geq \frac{1}{\alpha}
\]

where \( T_{mbo} \equiv \inf \{ t \mid \alpha \bar{e} ((1 + r)^t - 1) / r \geq K \},^{16} \) the first-best effort is achieved in equilibrium given strategies

\[
\sigma_t^e(E_{t-1}, C_{t-1}) = \begin{cases} \bar{e} & \text{if } (\forall s < t)e_s = \bar{e}, \\ 0 & \text{otherwise} \end{cases}
\]

\[
\sigma_t^c(E_{t-1}, C_{t-1}) = \begin{cases} 0 & \text{when } \beta_{t-1} < 1, \\ \leq m_t & \text{otherwise} \end{cases}
\]

\[
\sigma_t^l(E_t) = \begin{cases} 0 & \text{when } (\forall s \leq t)e_s = \bar{e} \text{ and } \beta_t < 1, \\ 1 & \text{otherwise} \end{cases}
\]

- Dispersed control can be supported by non-vested pension: If the manager’s pension is designed in such a way that a fixed amount of the manager’s income (in the first period) worth at least \( \bar{e} \) is withheld by the firm as the manager’s pension, the manager receives the interest the pension earns in each (subsequent) period, and the manager would lose the pension to a third party when leaving the firm, then the first-best effort can be achieved for all \( \alpha > 1 \).

**Proof:** The formal proof is given in the appendix. Here is the basic idea behind the proof of the result on compensation in ownership: in each period the manager weighs the gain and loss from shirking. If the gain is larger than the loss, the manager would shirk. Otherwise, the manager would not shirk. When the manager does not have enough wealth to purchase the firm’s assets, shirking would lead to the liquidation of the firm. According to the contract in (2), the gain (i.e. payoff) from choosing \( e_t < \bar{e} \) in period \( t \) would be the manager’s income minus effort in period \( t \beta_{t-1}(rK + \alpha e_t) + (1 - \beta_{t-1})\alpha \bar{e} - e_t \) plus income from the ownership of the firm on liquidation \( \beta_t K \). The loss (i.e. forgone income) from shirking is the income minus effort in period \( t \beta_{t-1}rK + \alpha \bar{e} - \bar{e} \) plus future income from the ownership \( \beta_t K \) plus future labor income from the firm minus effort, which is equal to the value of the firm to the manager \((\alpha - 1)\bar{e}/r \). The net benefit from shirking is \( nb_t \equiv \ldots \)

---

\( T_{mbo} \) is the earliest possible time when the manager can buy the firm’s assets after shirking. It would be incorrect to have the equality in the definition of \( T_{mbo} \), i.e., to have \( T_{mbo} = \inf \{ t \mid \alpha \bar{e} ((1 + r)^t - 1) / r \geq K \} \), because an equality means that the manager can buy the firm’s assets only without shirking.

\( T_{mbo} \) is the earliest possible time when the manager can buy the firm’s assets after shirking. It would be incorrect to have the equality in the definition of \( T_{mbo} \), i.e., to have \( T_{mbo} = \inf \{ t \mid \alpha \bar{e} ((1 + r)^t - 1) / r \geq K \} \), because an equality means that the manager can buy the firm’s assets only without shirking.

\( T_{mbo} \) is the earliest possible time when the manager can buy the firm’s assets after shirking. It would be incorrect to have the equality in the definition of \( T_{mbo} \), i.e., to have \( T_{mbo} = \inf \{ t \mid \alpha \bar{e} ((1 + r)^t - 1) / r \geq K \} \), because an equality means that the manager can buy the firm’s assets only without shirking.

\( l_t \) is set to be one when \( \beta_t \geq 1 \).
The manager would never shirk in this circumstance. The intuition behind this result is that when the manager does not have enough wealth to purchase the firm’s assets, shirking would lead to the liquidation of the firm and loss of future labor income from the firm. Because the future labor income from the firm (minus effort) is never smaller than the maximal possible effort saved $\bar{e}$, the manager never finds it profitable to shirk.

When the manager has enough wealth to take over the firm, shirking would lead to the takeover of the firm by the manager. In this case, the gain from choosing $e_t < \bar{e}$ in period $t$ is the manager’s income minus effort in period $t \beta_{t-1}(rK + \alpha e_t) + (1 - \beta_{t-1})\alpha \bar{e} - e_t$ plus future income from the ownership $\beta_t K$ plus the value of the firm to the manager $(\alpha - 1)\bar{e}$. The loss from shirking is the same as in the previous case. The net benefit from shirking is $NB_t = (\alpha \beta_{t-1} - 1)(e_t - \bar{e})$. When $\alpha \beta_{t-1} \geq 1$, $NB_t \leq 0$ for all $e_t \in [0, \bar{e}]$. When $\alpha \beta_{t-1} < 1$, $NB_t > 0$ for all $e_t \in [0, \bar{e})$. So the manager would shirk if and only if the ownership of the firm is smaller than $\frac{1}{\alpha}$ (i.e. when $\beta_{t-1} < \frac{1}{\alpha}$). The intuition behind this result is that when the manager has enough wealth to take over the firm, shirking would reduce the manager’s income from the project at the rate of $\alpha \beta_{t-1}$ and benefit the manager from the effort saved at the rate of one (which can be seen from the expression of $NB_t$). When the manager’s ownership of the firm is small, effort saved from shirking outweighs the decrease in income and the manager would benefit from shirking. When the manager’s ownership of the firm is large enough, shirking hurts the manager more than it benefits the manager and manager would not shirk.

Besides the condition in (4), two more conditions (given in (3)) also need to be satisfied. One condition is that the profitability of the project (as measured by $\alpha$) cannot be too low (i.e. $\alpha \geq 1 + r$). Otherwise, the manager’s benefit from shirking is always larger than the loss from shirking. The other condition is that the profitability of the project cannot be too high either (i.e. $\alpha < \frac{K}{\bar{e}}$). Otherwise, the manager is always better off shirking in the first period and then purchasing the firm’s assets with the income in the period.

The basic idea behind the result on the manager’s pension is that holding a pension worth at least $\bar{e}$ against shirking can prevent shirking by making shirking unprofitable for the manager. At the same time, having the pension withheld by the firm neither benefit nor hurt the manager financially because the present value of all future interest payments the manager receives from the pension is equal to the value of the manager’s pension itself. For the manager, having the pension withheld by the firm (and receiving the interest the pension earns) is equivalent to investing in a bond with infinite maturity.
Concentrated Control with an Aftermarket

In this subsection, we show that in the presence of an aftermarket, concentrated control cannot be supported with the threat to liquidate the project itself, but can be supported with both the threat to liquidate the project and gradual buyout of initial investment. The intuition behind this result is: When concentrated control is used, the investor can require the manager to buy out a (pre-specified) fraction of the initial investment in each period. If a large enough fraction of the initial investment is bought out early enough, the payment to the remaining part of the investment can be guaranteed by the liquidation value of the project. The quickest way to buy out all the initial investment is to ask the manager to spend all the income on initial investment buyout.

Same as the game in the previous subsection, the game in this subsection also depends on the contract that is in place, which is a standard debt contract with a debt repurchase provision. The contract specifies both the required interest payments \( \{\pi_t\} \) and the amount of debt-repurchase \( \{\Delta \psi_t K\} \) over time, where \( \Delta \psi_t \equiv \psi_t - \psi_{t-1} \) and \( \psi_t \) is the fraction of the debt issued at time 0 that has been retired by the end of period \( t \). In each period until liquidation of the firm, first the manager chooses effort \( e_t \), consumption \( c_t \), interest payment \( p_t \), and debt repurchase \( \phi_t K \). If both the interest payment and debt repurchase meet or exceed the amounts specified in the contract (i.e. \( p_t \geq \pi_t \) and \( \phi_t \geq \psi_t - \psi_{t-1} \)), the game continues in period \( t+1 \). Otherwise, the investor decides whether to liquidate the firm. In case liquidation occurs, the proceeds \( (K) \) from sale of the firm’s assets will be divided between the investor and the manager according to the absolute priority. The investor receives the value of the total outstanding debt \( (1-\psi_t)K \). The manager receives what is left after the investor is paid, which is \( \psi_t K \). If the manager has enough money \( (K) \) after consumption to buy the firm’s assets, the manager will do so in the aftermarket and then operate it without any incentive problems.

In the paper we consider a contract with the following interest payments and debt repurchase over time

\[
\begin{align*}
\pi_t &= (1-\psi_{t-1})rK \\
\psi_0 &= 0 \\
\psi_t &= \psi_{t-1}(1+r) + \frac{\alpha e}{K}
\end{align*}
\]

This contract requires that the manager spends all the income on debt repurchase until all the outstanding debt has been retired. It is possible that with this contract, the amount of debt retired by the manager may exceed 100% (i.e. \( \psi > 1 \)) at some
point over time. If that occurs, it means that the debt repurchase is complete and the investor needs to pay the manager the part that is above 100%.

The players’ strategies in the game are described by strategy functions $\sigma_t^e$ (for effort), $\sigma_t^c$ (for consumption as a fraction of available cash), $\sigma_t^p$ (for interest payment), $\sigma_t^t$ (for debt repurchased), and $\sigma_t^l$ (as liquidation indicator: 1 indicates liquidation and 0 indicates no liquidation). The solution concept for the game is subgame perfect Nash equilibrium.

Here is the manager’s choice problem at the start of the game. It is easy to write down the similar choice problem that corresponds to the subgame starting at any choice node.

**Problem 7 Manager’s problem (concentrated control with an aftermarket)** Given the contract terms $\{\pi_t\}_{t=1}^{\infty}$ and $\{\psi_t\}_{t=1}^{\infty}$ and the investor’s strategy $\{\sigma_t^I\}_{t=1}^{\infty}$ for liquidation, choose strategies $\sigma_t^e$ : $\mathbb{R}^{I(t-1)} \rightarrow [0, \bar{e}]$ for effort, $\sigma_t^c$ : $\mathbb{R}^{I(t-1)} \rightarrow [0, 1]$ for consumption, $\sigma_t^p$ : $\mathbb{R}^{I(t-1)} \rightarrow [0, rK + \alpha e_t]$ for interest payment, and $\sigma_t^t$ : $\mathbb{R}^{I(t-1)} \rightarrow [0, rK + \alpha e_t]$ for debt repurchase for each $t = 1, 2, ..., T_L \equiv \min\{t|t_t = 1\}$, to maximize

$$\sum_{t=1}^{T_L} \frac{c_t - e_t}{(1+r)^{t}} \frac{1}{(1+r)^{T_L}} \left( \frac{(\alpha - 1)\pi}{r} t(m_{T_L} \geq (1 - \psi_{T_L})K) + \psi_{T_L}K + m_{T_L} \right)$$

subject to:

$$m_0 = 0,$$

and for all $t = 1, 2, ..., T_L$:

$$e_t = \sigma_t^e(h_t^M),$$
$$c_t = (m_{t-1}(1 + r) + rK + \alpha e_t - p_t - \phi_t K)\sigma_t^c(h_t^M),$$
$$p_t = \sigma_t^p(h_t^M),$$
$$\phi_t = \sigma_t^\phi(h_t^M),$$
$$l_t = \sigma_t^l(h_t^I),$$
$$m_t = m_{t-1}(1 + r) + rK + \alpha e_t - p_t - \phi_t K - c_t \geq 0,$$
and

$$T_L = \min\{t|t_t = 1\}.$$ 

where $h_t^M \equiv (E_{t-1}, C_{t-1}, P_{t-1}, \Phi_{t-1})$ is the manager’s information set at $t$, and $h_t^I \equiv (E_t, P_t, \Phi_t)$ is the investor’s information set at $t$ with $E_t \equiv (e_1, e_2, ..., e_t)$, $C_t \equiv (c_1, c_2, ..., c_t)$, $P_t \equiv (p_1, p_2, ..., p_t)$, and $\Phi_t \equiv (\phi_1, \phi_2, ..., \phi_t)$.

**Problem 8 Investor’s problem (concentrated control with an aftermarket)** Given the contract terms $\{\pi_t\}_{t=1}^{\infty}$ and $\{\psi_t\}_{t=1}^{\infty}$ and the manager’s strategies $\{\sigma_t^e\}_{t=1}^{\infty}$ for ef-
for interest payment, and \( \{ \sigma^x_t \}_{t=1}^{\infty} \) for debt repurchase, choose strategies
\( \sigma^x_t : \mathbb{R}^m \to \{0, 1 - (p_t \geq \pi_t) \} \) for liquidation to maximize
\[
\sum_{t=1}^{T_L} p_t + \frac{\min\{K, (1 + r)(1 - \psi_{T_L-1})K - p_{T_L} - \phi T_k K\}}{(1 + r)^{T_L}} - K
\]
subject to:
\[
m_0 = 0,
\]
and for all \( t = 1, 2, ..., T_L:\)
\[
e_t = \sigma^e_t(h^M_t),
\]
\[
c_t = (m_{t-1}(1 + r) + \alpha e_t - p_t - \phi_t K)\sigma^c_t(h^M_t),
\]
\[
p_t = \sigma^p_t(h^M_t),
\]
\[
\phi_t = \sigma^\phi_t(h^M_t),
\]
\[
l_t = \sigma^l_t(h^l_t),
\]
\[
m_t = m_{t-1}(1 + r) + \alpha e_t - p_t - \phi_t K - c_t \geq 0, \text{ and}
\]
\[
T_L = \min\{t|l_t = 1\}.
\]

We have the following result on concentrated control.

**Theorem 5  Equilibrium (concentrated control with an aftermarket)** In the model of concentrated control with an aftermarket, whether the first-best can be achieved depends both on how large the effectiveness of effort \( \alpha \) is and on whether gradual buyout of initial investment is used. Specifically:

- Concentrated control cannot be supported by the threat to liquidate the project itself: When debt repurchase is not used, the first-best effort cannot be achieved, and in fact no equilibrium has positive effort along the equilibrium path.

- Concentrated control can be supported for projects with infinite life by debt repurchase: When the contract in (7) is used and when

\[
1 + r^2 K/\tau \leq \alpha < (1 - r)K/\tau,
\]

the first-best effort is achieved in equilibrium given strategies

\[
\sigma^e_t(h^M_t) = \tau \text{ for all } t
\]
\[
\sigma^c_t(h^M_t) = \begin{cases} 0 & \text{when } \psi_{t-1} < 1 \\ \leq m_t & \text{otherwise} \end{cases}
\]
\[
\sigma^p_t(h^M_t) = \begin{cases} rK & \text{if } (\forall s < t) \ p_s \geq \pi_s \text{ and } \phi_s \geq \psi_s - \psi_{s-1} \\ 0 & \text{otherwise} \end{cases}
\]
\[
\sigma^\phi_t(h^M_t) = \begin{cases} (\psi_{t-1} - \psi_{t-1})K & \text{if } (\forall s < t) \ p_s \geq \pi_s \text{ and } \phi_s \geq \psi_s - \psi_{s-1} \\ 0 & \text{otherwise} \end{cases}
\]
and

\[ l_t(h^t_i) = \begin{cases} 
0 & \text{when } (\forall s \leq t) \ p_s \geq \pi_s \text{ and } \phi_s \geq \psi_s - \psi_{s-1} \\
1 & \text{otherwise}
\end{cases} \]

**PROOF:** The formal proof is given in the appendix. Here is the basic idea behind the proof: in each period the manager weighs the gain and loss from not making the interest payment (as well as not buying back the debt). If the gain is larger than the loss, the manager would not make the interest payment. Otherwise, the manager would make the interest payment. Since the manager can never have enough cash to purchase the firm’s asset (i.e. \( m_t < K \)) before all the outstanding debt is retired by (7) and (8), we need to consider only whether the manager has enough cash to purchase the firm’s outstanding debt, which has a value of \( d_t \equiv (1 + r)(1 - \psi_{t-1})K \) in period \( t \). When the manager does not have enough cash to purchase the firm’s debt, not making the interest payment would lead to the liquidation of the firm. According to the debt contract in (7), the gain (i.e. payoff) from not making the interest payment (as well as not buying back any debt) in period \( t \) is the manager’s income minus effort in the period \( rK + \alpha \bar{e} - \bar{e} \) plus what is left to the manager after liquidation of the firm \( \max\{K - d_t, 0\} = \max\{(1 + r)\psi_{t-1}K - rK, 0\} \). The loss (i.e. foregone income) from doing so is (the manager’s cash income after the interest payment and debt repurchase in the period (which is zero) plus) the manager’s disutility of effort \(-\bar{e} \) in period \( t \) plus the part of the debt that has been retired by the end of period \( t \) \( \psi_tK = (1 + r)\psi_{t-1}K + \alpha \bar{e} \) plus the value of the firm to the manager \((\alpha - 1)\bar{e}/r \).

The net benefit from not making the interest payment is \( nb_t \equiv rK + \alpha \bar{e} - \bar{e} + \max\{K - d_t, 0\} + \bar{e} - \psi_tK - (\alpha - 1)\bar{e}/r \). When \((1 + r)\psi_{t-1} \geq r \), the net benefit becomes \( nb_t = - (\alpha - 1)\bar{e}/r < 0 \). When \((1 + r)\psi_{t-1} < r \), the net benefit is \( nb_t = rK + (1 + r)\psi_{t-1}K - (\alpha - 1)\bar{e}/r \leq rK - (\alpha - 1)\bar{e}/r \leq 0 \) by (8). Since the net benefit is always nonpositive, the manager will never miss the interest payment in this circumstance. The intuition behind this result is that when the manager does not have enough cash to purchase all the outstanding debt, not making the interest payment would lead to the liquidation of the firm and loss of future income from the firm. Because the future income from the firm is never smaller than the maximum possible interest payment saved \( rK \), the manager never finds it profitable to miss the interest payment.

When the manager has enough cash to purchase the firm’s the outstanding debt, i.e., when \( m_t \geq d_t \), not making the interest payment would lead to the takeover of the project by the manager. At this time we have \( d_t \leq m_t < K \) because \( m_t \) is always smaller than \( K \). The gain from not making the interest payment in period \( t \) is the manager’s income minus effort in period \( t rK + \alpha \bar{e} - \bar{e} \) minus the payment that needs to be made in order to take over the firm \( \min\{K, d_t\} = d_t \) plus all future cash flows minus effort from the firm \( K + (\alpha - 1)\bar{e}/r \). The loss from not making the interest payment is the same as in the previous situation. The net benefit from not making the
interest payment is \( NB_t \equiv rK + \alpha \bar{e} - \bar{e} - d_t + K + (\alpha - 1)\bar{e}/r + \bar{e} - \psi_t K - (\alpha - 1)\bar{e}/r = 0 \), which implies that the manager can never benefit from not making the interest payment. The intuition behind this result is that when the manager has enough cash to purchase the firm’s outstanding debt, the value of the debt is smaller than the value of the firm’s assets \( K \) (i.e. \( d_t < K \)). In this case, the debt payment is guaranteed by the liquidation value of the project and has always to be made in full: it will either come from the manager or from the proceeds from the liquidation of the project. In this circumstance, the manager cannot benefit from not making the interest payment and would always make the interest payment. (Explanations for the necessity of (8) are similar to those for the necessity of (3) in the proof of Theorem 4 and are omitted here.)

Based on the results obtained above, we can draw the following conclusions on the impact of an aftermarket on incentives and on financing of the project:

- Just adding an aftermarket destroys incentives.
- Incentives can always be restored with non-vested pensions.
- Incentives can also be restored with compensation in ownership and gradual buyout of initial investment when potential economic profits are neither very small nor very large:
  1. Dispersed control is preferred when production is capital-intensive.
  2. Concentrated control is preferred when production is effort-intensive.

4 Conclusion

This paper studies how the assignment of operational control rights can be used to reduce potential economic inefficiency. The analysis focuses on two alternative institutional arrangements with different operational control rights: one offers concentrated control and the other offers dispersed control. We find that the two arrangements create different potential incentive problems: With concentrated control, the manager may take the entire cash flow out of the firm and with dispersed control, the manager may expend too little effort. Depending on the relative severity of the two incentive problems, either arrangement may dominate the other. The existence of an aftermarket in the firm’s assets after the firm is dissolved undermines the manager’s incentives while still with the firm. Facing an aftermarket, incentives can be maintained by institutions such as sinking funds, or deferred compensation such as non-vested pensions or nontrading stock compensation.
References


Appendix: Proofs of the Theorems

Proof of Theorem 1: (i) Given \((\alpha - 1)\sigma/r \geq \sigma\), we need to show that the posited contract and strategies implement the first-best.
On the equilibrium path, the firm is never liquidated and the manager extends full effort $\bar{e}$ in each period and is paid the marginal product $\bar{e}\alpha$. This gives the manager surplus of $\bar{e}((\alpha - 1))$ in each period, valued in total at $\bar{e}((\alpha - 1))/r$. The investor gets total surplus of $-K$ at time 0 and then $rK + \alpha\bar{e} - \alpha\bar{e} = rK$ per period valued in total as $-K + rK/r = 0$. Since the sum of the two agents’ surplus equals the first-best total surplus, we know this choice will be the best possible once we show that it is an equilibrium as claimed.

To verify that the claimed strategies are an equilibrium given the constant wage $w_t = \bar{e}\alpha$, we have to show that the posited strategy is a Nash equilibrium given any hypothetical history.

First, consider optimality of the strategies of the manager. If the history has only full effort $e_s = \bar{e},^{18}$ the manager collects surplus of $\bar{e}((\alpha - 1))$ per period by following the strategy (given the posited response by the investor). Any deviation from this strategy by the manager results in a net gain by the manager of the reduction of effort (no more than $\bar{e}$) in that period and a loss of all surplus (present value $\bar{e}((\alpha - 1))/r$) in future periods. Since $\alpha > 1 + r$ (by assumption in (i)), the loss from deviating is greater than the gain, confirming that the posited strategy is optimal given an history with maximal effort. If the history has any non-optimal effort, then any effort by the manager is wasted because the salary does not depend on effort and the posited strategy has the investor liquidating independent of what effort the manager chooses.

Now, consider optimality of the strategies of the investor. If the history has only full effort $e_s = \bar{e}$, the investor receives $rK$ per period forever by following the strategy. A deviation would trade this cash flow for of $rK$ at each subsequent date for $K$ at the date of liquidation, which is an even trade. Therefore, following the strategy is (weakly) optimal. After a history including at least one instance of suboptimal effort, the manager is expected to spend zero effort in all future periods. Given the manager’s posited strategy, the investor expects receive output of just $rK$ and spend wage of $\bar{e}\alpha$ each period. This would be a fair return on the capital $K$ absent the wage, but is a subnormal return altogether. Consequently, the investor is best off liquidating as soon as possible, which confirms the optimality of the strategy. This finishes the confirmation that the claimed strategies are in subgame perfect Nash equilibrium.

(ii) For $(\alpha - 1)\bar{e}/r < \bar{e}$, we need to show that any contract and strategies that implement an equilibrium with nonnegative surplus for both agents must also have zero effort and zero surplus for both agents.

Let $\{w_t\}$ be the wages promised under the contract, let $T_L$ be the liquidation date along such a path (could be $\infty$) and let $(e_1, ..., e_{T_L})$ be the equilibrium efforts. Since the investor does not want liquidation at time $t < T_L$, it follows that $K \leq K/(1 + \ldots)

---

18 As of the initial node, there has been no deviation, so the initial node is covered by this case.
\[ r^{T_L-t} + \sum_{s=t+1}^{T_L} (rK + \alpha e_t - w_t)/(1 + r)^{s-t} \] or equivalently

\[
(11) \quad \sum_{s=t+1}^{T_L} \frac{w_t}{(1 + r)^{s-t}} \leq \sum_{s=t+1}^{T_L} \frac{\alpha e_t}{(1 + r)^{s-t}},
\]

and the same formula follows for \( t = 0 \) because of the assumption of nonnegative surplus. However, given that \( \alpha < 1 + r \), we will show that this implies that the manager must want to deviate at some time less than \( T_L \). Letting \( e_{sup} \equiv \sup_{t \in \{1,2,\ldots,T_L\}} e_t \) (which exists because the \( e_t \)'s are all bounded above by \( \bar{e} \)), we have that

\[
e_t \leq \sum_{s=t+1}^{T_L} \frac{w_s - e_s}{(1 + r)^{s-t}} \leq \sum_{s=t+1}^{T_L} \frac{\alpha e_s - e_s}{(1 + r)^{s-t}} \leq \sum_{s=t+1}^{\infty} \frac{e_{sup}(\alpha - 1)}{(1 + r)^{s-t}} = e_{sup} \frac{(\alpha - 1)}{r}.
\]

This says that the gain from shirking (\( e_t \)) is no greater than the bound \( e_{sup} \frac{(\alpha - 1)}{r} \) on the loss from shirking. If \( e_{sup} = 0 \), we are done since (11) and nonnegativity of the \( e_t \)'s and \( w_t \)'s imply that all \( e_t \)'s, \( w_t \)'s are zero so that all surplus is zero. If \( e_{sup} > 0 \), there is a contradiction since \((\forall t) e_t \leq e_{sup}(\alpha - 1)/r \Rightarrow e_{sup} \leq e_{sup}(\alpha - 1)/r \Rightarrow \alpha \geq 1 + r. \) Therefore, we are done.

**Proof of Theorem 2:** (i) Given \( (\alpha - 1)\bar{e}/r \geq \bar{e} \), we need to show that the posited contract and strategies implement the first-best.

On the equilibrium path, the manager extends full effort and pays the investor the promised amount \( p_t = \pi_t = rK \) in each period. Since the promise is always honored, the manager never has an option to liquidate along the equilibrium path, and is therefore optimizing by choosing \( l_t = 0 \), the only feasible value, for all \( t \). The investor’s expected payoff is \( \sum_{t=1}^{\infty} rK/(1 + r)^t - K = 0 \). For the manager, an alternative strategy along the equilibrium path can have effort less than \( \bar{e} \) and/or a payment different from \( rK \) in one or more periods. The first deviation from the payment of \( rK \) results in liquidation by the investor the next period, saving at most \( rK \) this period but losing surplus of \((\alpha - 1)\bar{e} \) in all future periods. The surplus lost has present value \((\alpha - 1)\bar{e}/r \) which we are assuming to be at least as large as \( rK \) which is the loss. Therefore, changing the payment strategy cannot increase value. Reducing the
effort, whether in addition to a change in payments or without a change in payments, cannot increase value because the period felicity \( c_t = rK + \alpha e_t - p_t - e_t \) is increasing in \( e_t \) and because changing effort this period does not affect the subsequent evolution of the game.

Off the equilibrium path, if there has been a deviation in payment, the investor should optimally liquidate because the investor expects no payment \( (p_t = 0) \) in all future periods. Similarly, if there has been a deviation in payment, the manager expects liquidation whatever payment is chosen, and may as well choose payment 0. If off the equilibrium path and all deviations have been in effort, the subgame and the posited continuation are the same as on the equilibrium path, so the posited strategies are optimal as shown above.

(ii) Given that \( (\alpha - 1)e/r < rK \) we need to show that no equilibrium has nonnegative payoffs for both the manager and the investor. Suppose to the contrary that there is such an equilibrium, with efforts \( \{e_t\} \), payments \( \{p_t\} \), liquidation policy \( \{l_t\} \), and liquidation date \( T_L \equiv \min\{t|t_t = 1\} \) (recall \( T_L \equiv \infty \) if all \( l_t = 0 \)). Nonnegative payoff for the investor says that

\[
\sum_{t=1}^{T_L} p_t (1 + r)^t \geq \sum_{t=1}^{T_L} rK (1 + r)^t.
\]

Since we have an equilibrium, it must not make the manager better off (given the investor’s policy) to switch to the strategy choosing payoff 0 at all times and the equilibrium effort \( e_t \) up through time \( T_L \) and maximum effort \( \tau \) thereafter if the firm is still operating. (This is an unconditional strategy that does not depend on the history: this is not intended to be the best response along the equilibrium path or in subgames, just a feasible strategy.) Let \( \tau = \min\{t \leq T_L|p_t > 0\} \). We know \( \tau < \infty \) from the participation constraint (12). Up to time \( \tau \), the game goes the same as in the original equilibrium. At time \( \tau \), the manager chooses payoff \( p_\tau = 0 \) while the original game chooses \( p_\tau > 0 \). This is a savings of \( p_\tau \) (or \( p_\tau/(1 + r)^\tau \) in time 0 present value), but may result in a loss of all future rents. Given that the new strategy never pays out a positive amount, the manager’s consumption is zero after the firm is liquidated or the nonnegative quantity \( rK + (\alpha - 1)e_t - p_t = rK + (\alpha - 1)e_t \) until the firm is liquidated. As a result, the most the manager can lose is the equilibrium payoff after \( \tau \) (and will lose less if the positive payoff is realized in any period after \( \tau \)). Since this deviation from the strategy cannot make the manager better off than the equilibrium strategy, it must be that

\[
\frac{p_\tau}{(1 + r)^\tau} \leq \sum_{t=\tau+1}^{T_L} \frac{rK + (\alpha - 1)e_t - p_t}{(1 + r)^t},
\]

31
where the right-hand side gives the time zero present value of the manager’s payoff from times after \( \tau \). Bringing all the payoff terms to the left-hand side, we can rewrite (13) as

\[
(14) \quad \sum_{t=\tau}^{T_L} \frac{pt}{(1 + r)^t} \leq \sum_{t=\tau+1}^{T_L} \frac{rK + (\alpha - 1)e_t}{(1 + r)^t}.
\]

By definition of \( \tau \), \( pt = 0 \) for all \( t < \tau \), so that \( \sum_{t=1}^{T_L} pt/(1 + r)^t = \sum_{t=\tau}^{T_L} pt/(1 + r)^t \). Therefore, we can combine (12) and (14) and multiply by \( (1 + r)^\tau \) to obtain

\[
\begin{align*}
\frac{rK}{(1 + r)^{t+1}} &\leq \sum_{t=\tau}^{T_L} \frac{(\alpha - 1)e_t}{(1 + r)^{t+1}} - \sum_{t=1}^{\tau-1} \frac{rK}{(1 + r)^{t-\tau}} \\
&\leq \sum_{t=\tau}^{T_L} \frac{(\alpha - 1)e}{(1 + r)^{t+1}} \\
&\leq \sum_{t=\tau}^{\infty} \frac{(\alpha - 1)e}{(1 + r)^{t+1}} \\
&= \frac{(\alpha - 1)e}{r},
\end{align*}
\]

contradicting \( (\alpha - 1)e/r < rK \).}

**Proof of Theorem 4:** (i) It suffices to show that the investor’s strategy for dismissal in (6) itself cannot deter the manager from shirking when neither compensation in ownership nor non-vested pension is used. (In this case, we have that \( \beta \) is equal to zero before the takeover (if any) and is equal to one after the takeover.) One way to prove this is to show that given the investor’s strategy for dismissal in (6), the manager is better off playing the following strategy on effort and consumption

\[
(15) \quad e_t(E_{t-1}, C_{t-1}) = \begin{cases} 
\bar{e} & \text{when } \beta_{t-1} = 0 \text{ and } m_t < K \\
0 & \text{when } \beta_{t-1} = 0 \text{ and } m_t \geq K \\
\bar{e} & \text{when } \beta_{t-1} = 1 \\
\end{cases}
\]

\[
c_t(E_{t-1}, C_{t-1}) = \begin{cases} 
0 & \text{when } \beta_{t-1} = 0 \\
\leq m_t & \text{when } \beta_{t-1} = 1.
\end{cases}
\]

which involves shirking (when the manager is able to take over the firm’s assets) and buying the firm’s assets in the aftermarket (whenever the manager has enough wealth to do so), than playing the strategy in (5).

32
Given the investor’s strategy for dismissal, the manager’s expected payoff from playing the strategy in (5) is \((\alpha \bar{e} - \bar{\tau})/r\). If instead playing the strategy in (15), the manager gets \((\alpha \bar{e} - \bar{\tau})/r + \bar{\tau}/(1 + r)^{T_L}\) where \(T_L \equiv \inf\{t | l_t = 1\} = \inf\{t | m_t \geq K\}\). The gain for the manager equals \(\bar{\tau}/(1 + r)^{T_L} > 0\). Shirking in period \(T_L\) would lead to both shirking and liquidation in periods before \(T_L\).

(ii) We prove only the optimality of the manager’s strategy along the equilibrium path for given investor’s strategy. Proofs of the optimality of the manager’s strategy on off-equilibrium paths and optimality of investor’s strategy are similar to those in proofs of Theorem 1 and Theorem 2.

First, we show that it is necessary to have \(1 + r \leq \alpha \leq K/\bar{e}\). The manager’s payoff from shirking in the first period is the one-period wage \(\alpha \bar{e}\). The manager’s payoff from always exerting the first-best effort is \(\alpha \bar{e} - \bar{e} + (\alpha - 1)\bar{e}/r\). The manager will not shirk when \(\alpha \geq 1 + r\) and will shirk otherwise. It is quite clear that we need \(\alpha < K/\bar{e}\) to hold. Otherwise, the manager is guaranteed to own the entire firm by the share grant.

Now we prove that when (3) and (4) hold, for given investor’s strategy in (6), the manager cannot profit from a deviation from strategies in (5). Given the investor’s strategy for liquidation, the manager’s expected payoff from the strategy in (5) is \(\sum_{t=1}^{\infty} (\alpha \bar{e} - \bar{e})/(1 + r)^t = (\alpha \bar{e} - \bar{e})/r\). Now consider a deviation from the maximal effort. Suppose the manager chooses \(e_t < \bar{e}\) in period \(t \geq 1\). If the manager does not have enough wealth to purchase the firm’s assets, i.e., if \(m_t < (1 - \beta_t)K\), this strategy would lead to the liquidation of the project. The manager’s expected payoff from playing this new strategy is

\[
\frac{\beta_t K + \kappa_t - e_t}{(1 + r)^t} - \sum_{i=1}^{t-1} \frac{\bar{e}}{(1 + r)^i} = \frac{\beta_{t-1} K + (1 - \beta_{t-1})\alpha \bar{e} + \beta_{t-1}(rK + \alpha e_t) - e_t}{(1 + r)^t} - \sum_{i=1}^{t-1} \frac{\bar{e}}{(1 + r)^i} = \sum_{i=1}^{t-1} \frac{\alpha \bar{e} - \bar{e}}{(1 + r)^i} + \frac{(1 - \beta_{t-1})\alpha \bar{e} + \beta_{t-1}\alpha e_t - e_t}{(1 + r)^t}. 
\]

The change in expected payoff is, when \(\alpha \beta_{t-1} \geq 1\),

\[
\frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)e_t}{(1 + r)^t} - \sum_{i=t}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^i} \leq \frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)\bar{e}}{(1 + r)^t} - \sum_{i=t}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^i}.
\]
\[
\begin{align*}
&= \frac{(\alpha - 1)\bar{e}}{(1 + r)^t} - \sum_{i=t}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^i} \\
&= - \sum_{i=t+1}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^i} < 0.
\end{align*}
\]

The change in expected payoff is, when \(\alpha \beta_{t-1} < 1\),

\[
\begin{align*}
&= \frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)e_t}{(1 + r)^t} - \sum_{i=t}^{\infty} \frac{(\alpha - 1)\bar{e}}{(1 + r)^i} \\
&\leq \frac{(1 - \beta_{t-1})\alpha \bar{e}}{(1 + r)^t} - \frac{\alpha \bar{e} - \bar{e}}{r(1 + r)^{t-1}} \\
&= \frac{\bar{e}}{r(1 + r)^t} (1 + r - \alpha - \alpha \beta_{t-1}r) \leq 0
\end{align*}
\]

by assumptions \(\alpha \geq 1 + r\) and \(\beta_{t-1} \geq 0\). So the deviation does not pay.

If the manager has enough wealth to purchase the firm’s assets, i.e., if \(m_t \geq (1 - \beta_t)K\), this strategy would lead to the takeover of the firm by the manager. The manager’s expected payoff from playing this new strategy is

\[
\begin{align*}
&= \frac{\beta_tK + \kappa_t - e_t}{(1 + r)^t} + \sum_{i=t+1}^{\infty} \frac{rK + \alpha \bar{e} - \bar{e}}{(1 + r)^i} - \frac{K}{(1 + r)^t} - \sum_{i=1}^{t-1} \frac{\bar{e}}{(1 + r)^i} \\
&= \frac{\beta_{t-1}K + (1 - \beta_{t-1})\alpha \bar{e} + \beta_{t-1}(rK + \alpha e_t) - e_t}{(1 + r)^t} \\
&\quad + \sum_{i=t+1}^{\infty} \frac{\alpha \bar{e} - \bar{e}}{(1 + r)^i} - \sum_{i=1}^{t-1} \frac{\bar{e}}{(1 + r)^i} \\
&= \sum_{i=1}^{\infty} \frac{\alpha \bar{e} - \bar{e}}{(1 + r)^i} + \frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)e_t - (\alpha - 1)\bar{e}}{(1 + r)^t}
\end{align*}
\]

The change in expected payoff is, when \(\alpha \beta_{t-1} \geq 1\),

\[
\begin{align*}
&= \frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)e_t - (\alpha - 1)\bar{e}}{(1 + r)^t} \\
&\leq \frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)\bar{e} - (\alpha - 1)\bar{e}}{(1 + r)^t} = 0.
\end{align*}
\]
The deviation does not pay. The change in expected payoff when $\alpha \beta_{t-1} < 1$, however, is

$$\frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)e_r - (\alpha - 1)\bar{e}}{(1 + r)^t} \geq \frac{(1 - \beta_{t-1})\alpha \bar{e} + (\alpha \beta_{t-1} - 1)e_r - (\alpha - 1)\bar{e}}{(1 + r)^t} = 0.$$

The manager can benefit from this deviation.

**Proof of Theorem 5:** Here we provide only proofs of (ii) and (iii). The proof of (i) is similar to that in the proof of Theorem 4.

(ii) We prove only the optimality of the manager’s strategy along the equilibrium path for given investor’s strategy. Proofs of the optimality of the manager’s strategy on off-equilibrium paths and optimality of investor’s strategy are similar to those in proofs of Theorem 1 and Theorem 2.

First, it is quite clear that we need $\alpha \geq 1 + r^2K/\bar{e}$ or equivalently $rK \leq (\alpha - 1)\bar{e}/r$ to hold. Otherwise, the manager is better off making zero interest payment in the first period. It is also clear that we need $\alpha < (1 - r)K/\bar{e}$ to hold. Otherwise, the manager will make zero interest payment and purchase the firm’s assets with the income in the first period.

Now we prove that for all $\alpha \in [1 + r^2K/\bar{e}, (1 - r)K/\bar{e})$ and for given investor’s strategy in (10), the manager cannot profit from a deviation from strategies in (9). Given the investor’s strategy for liquidation, the manager’s expected payoff from the strategy in (9) is $\sum_{t=1}^{\infty} (\alpha \bar{e} - \bar{e})/(1 + r)^t = (\alpha \bar{e} - \bar{e})/r$. Now we consider a deviation from the strategy in (9), which would consist of a deviation from the interest payment and a possible deviation from the required debt repurchase (since a deviation from the maximal effort would only hurt the manager and a deviation from zero consumption is ruled out here). Suppose the manager chooses a lower interest payment $p_t < \pi_t$ and a possibly lower debt repurchase $\phi_t \leq \pi_t$ in period $t \geq 1$. The cash the manager has and the value of the outstanding debt are

$$m_t = rK + \alpha \bar{e} - p_t - \phi_t K$$
$$d_t = (1 + r)(1 - \psi_{t-1})K - p_t - \phi_t K$$

respectively.

If $m_t < \min\{K, d_t\}$, i.e., if the manager does not have enough cash to purchase the outstanding debt, this deviation would lead to liquidation of the project. The
The manager's expected payoff from playing this new strategy is

\[
\frac{m_t + \max\{K - d_t, 0\}}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s}
\]

\[
rK + \alpha \bar{c} - \bar{c} - p_t - \phi_t K
\]

\[
+ \max\{K - ((1 + r)(1 - \psi_{t-1})K - p_t - \phi_t K), 0\} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s}
\]

\[
rK + \alpha \bar{c} - \bar{c} - p_t - \phi_t K
\]

\[
+ \max\{K + p_t + \phi_t K - (1 + r)(1 - \psi_{t-1})K, 0\} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s}
\]

\[
rK + \alpha \bar{c} - \bar{c} - p_t - \phi_t K
\]

\[
+ \max\{\psi_{t-1}(1 + r)K - rK + p_t + \phi_t K, 0\} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s}
\]

The change in expected payoff is, when \( \psi_{t-1}(1 + r)K + p_t + \phi_t K \geq rK \),

\[
\frac{\alpha \bar{c} - \bar{c} + \psi_{t-1}(1 + r)K}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s} - \sum_{s=1}^{\infty} \frac{\alpha - 1 \bar{c}}{(1 + r)^s}
\]

\[
= \frac{(\alpha - 1)\bar{c}}{(1 + r)^t} + \sum_{s=1}^{t-1} \frac{\alpha \bar{c}}{(1 + r)^s} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s} - \sum_{s=1}^{\infty} \frac{\alpha - 1 \bar{c}}{(1 + r)^s}
\]

\[
= \sum_{s=1}^{t} \frac{(\alpha - 1)\bar{c}}{(1 + r)^s} - \sum_{s=1}^{\infty} \frac{(\alpha - 1 \bar{c})}{(1 + r)^s}
\]

\[
= - \sum_{s=t+1}^{\infty} \frac{(\alpha - 1 \bar{c})}{(1 + r)^s} < 0.
\]

The change in expected payoff is, when \( \psi_{t-1}(1 + r)K + p_t + \phi_t K < rK \),

\[
\frac{rK + \alpha \bar{c} - \bar{c} - p_t - \phi_t K}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s} - \sum_{s=1}^{\infty} \frac{\alpha - 1 \bar{c}}{(1 + r)^s}
\]

\[
\leq \frac{rK + \alpha \bar{c} - \bar{c} - p_t - \phi_t K}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s} - \sum_{s=1}^{\infty} \frac{\alpha - 1 \bar{c}}{(1 + r)^s}
\]

\[
\leq \frac{(\alpha - 1)\bar{c}}{r(1 + r)^t} + \frac{(\alpha - 1)\bar{c}}{(1 + r)^t} - \sum_{s=1}^{t-1} \frac{\bar{c}}{(1 + r)^s} - \sum_{s=1}^{\infty} \frac{\alpha - 1 \bar{c}}{(1 + r)^s}
\]

36
\[
\begin{align*}
&= \frac{(\alpha - 1) \bar{e}}{r(1+r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} - \sum_{s=1}^{t-1} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} - \sum_{s=t+1}^{\infty} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} \\
&= \frac{(\alpha - 1) \bar{e}}{r(1+r)^t} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} - \sum_{s=1}^{t-1} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} - \frac{\bar{e}}{r(1+r)^t} \\
&= -\sum_{s=1}^{t-1} \frac{\alpha \bar{e}}{(1+r)^s} < 0.
\end{align*}
\]

(since by assumption \(rK \leq \frac{(\alpha-1)\bar{e}}{r}\) or equivalently \(\alpha \geq 1 + \frac{r^2K}{\bar{e}}\)). So the deviation does not pay.

Now we consider the situation in which the manager has enough cash to take over the firm, i.e., \(m_t \geq \min\{K, d_t\}\). From \(\alpha < (1-r)K/\bar{e}\), we have \(m_t = rK + \alpha \bar{e} - p_t - \phi_t K \leq rK + \alpha \bar{e} < K\). These two inequalities imply that \(d_t < K\) always holds, which in turn implies \(\min\{K, d_t\} = d_t\). The manager’s expected payoff from playing this new strategy is

\[
\begin{align*}
&= \frac{m_t}{(1+r)^t} + \sum_{s=t+1}^{\infty} \frac{rK + (\alpha - 1) \bar{e}}{(1+r)^s} - \min\{K, d_t\} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} \\
&= \frac{(1+r)K + \alpha \bar{e} - \bar{e} - p_t - \phi_t K}{(1+r)^t} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} \\
&= \frac{(1+r)\psi_{t-1} K + \alpha \bar{e} - \bar{e}}{(1+r)^t} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} \\
&= \sum_{s=1}^{t-1} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} + \sum_{s=t+1}^{\infty} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} - \sum_{s=1}^{t-1} \frac{\bar{e}}{(1+r)^s} \\
&= \sum_{s=1}^{\infty} \frac{(\alpha - 1) \bar{e}}{(1+r)^s}
\end{align*}
\]

The change in expected payoff is

\[
\sum_{s=1}^{\infty} \frac{(\alpha - 1) \bar{e}}{(1+r)^s} - \sum_{s=1}^{\infty} \frac{\bar{e}}{(1+r)^s} = 0.
\]

So the deviation does not pay.