Debt and Equity*

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Abstract

Which is better for a small firm: debt financing or equity financing? A simple intertemporal model suggests that the two have very different potential incentive problems. With equity financing, the manager (an employee) may expend too little effort, while with debt financing, the manager (the owner) may keep the entire cash flow and default on the debt. Depending on the relative severity of these two incentive problems, either debt or equity may dominate the other. These problems are exacerbated by the possibility of an MBO and are reduced by sinking funds, stock or option compensation and non-vested pensions.

Comments are welcome

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1 Introduction

The conflict between a manager and outside financial claimants is a well-known source of economic inefficiency. In a proprietorship with outside debt, the conflict of interest may induce the owner-manager to take assets out of the firm or to choose too much risk, in either case collecting value at the expense of the bondholders. In a corporation with outside equity, an employee-manager may have the incentive to expend too little effort or to take on projects that please the manager but do not add value for shareholders. It is our goal to explore what different incentive problems imply for the efficiency of different organizational forms. We explore these issues in a simple intertemporal model with a stark choice between a proprietorship raising money only through debt and a simple corporate form with outside equity and a fixed wage for the employee-manager. We find that the two forms have very different incentive problems, and either can be more efficient depending on the relative severity of the incentive problems. Allowing the possibility of an MBO tends to erode incentives in both forms, since the manager can “trash” the firm and buy it at a discount. Institutions such as employee share ownership, sinking funds and non-vested pensions can ameliorate the incentive problems both for the corporate form and for the proprietorship.

For the corporate form, the manager and equityholder both know the effort level. The effort is certainly known to the employee-manager who chose it, and the equityholder can infer effort from the output, because there is no uncertainty in our model. However, effort is assumed not to be publicly observable, which rules out the use of direct incentive contracts for the manager’s compensation. We can think of this as a result of the absence of accounting systems to record the output or the manager’s ability to divert the output for private benefits (so the true output is not observable). In a simple firm, perhaps output is collected by the equityholder, like cash at the door of a night club or a crop planted and cared for by the manager but harvested by the equityholder. At the outset, the manager signs a wage contract that promises a fixed wage in each period. The equityholder is obligated to pay the fixed wage for as long as the manager is in charge of the project, but has full discretion to fire the manager at any time. Given that the manager’s wage does not depend on the output, the manager may have an incentive to expend no effort and just collect the wage. Depending on the parameter values, the manager’s shirking may or may not be deterred by the threat of dismissal.

For the proprietorship, the manager is also the equityholder of the firm. In this case, the owner-manager has no incentive to shirk, but may have an incentive to take assets out of the firm and default on the debt. Depending on
the parameter values, removal of assets followed by default may or may not be deterred by a threat to liquidate the firm when the interest payment is missed.

Whether the corporate form or the proprietorship is better depends on the relative severity of their incentive problems, which depends in turn on such determinants as the size of the initial investment, the future profitability of the project, how difficult effort is for the manager and the interest rate. For the corporate form, an increase in the employee-manager’s difficulty of effort or a decrease in the project’s future profitability increases the manager’s benefit from shirking, and makes the employee-manager’s incentive problem more severe. For the proprietorship, an increase in the interest rate or a decrease in the project’s future profitability increases the owner-manager’s benefit from defaulting, and therefore makes the owner-manager’s incentive problem more severe.

The possibility of an MBO may jeopardize funding of the project, because investors may be concerned the manager will expend too little effort and then purchase the firm at a discount. In other words, firing the manager or liquidating the project is not a punishment when an MBO is possible, because the manager can take over the firm and reinstate the project right after the dismissal or liquidation.

Other institutions can be used to mitigate the incentive problems and support the financing of the project. Debt financing can be supported by sinking funds. Equity financing can be supported by employee share ownership or managers’ non-vested pensions. The intuition behind using sinking funds to support debt financing is that sinking funds can be used to retire debt. If a large enough proportion of the debt is retired early enough, the payment of the remaining debt can be guaranteed by the liquidation value of the project. The intuition behind using employee share ownership to support equity financing is that compensating the manager in the company’s shares transfers the ownership of the firm from the investor to the manager and can eliminate the manager’s incentive to shirk. The intuition behind using managers’ non-vested pensions to support equity financing is that managers’ non-vested pensions can be held as collateral against the shirking. An important issue, which we have to resolve when using employee share ownership, is to find a feasible way to divide the firm’s output between the two parties (according to the fractions of their ownership of the firm) when the output itself is not publicly verifiable. We show that the division of the output between the two parties can be fairly easily done for some projects.

Our model has two predictions on the two organizational forms. One prediction is that the corporate form is more likely to be used by non-effort-intensive firms and the proprietorship is more likely to be used by non-capital-intensive
firms, whatever industries these firms may operate in. The other prediction is that it can be more difficult and tedious to arrange equity financing than to arrange debt financing, because of certain requirements that need to be satisfied in equity financing, such as the investor’s direct involvement in the operation of the firm. As a result, debt may be the preferred choice of the two when both can be used, and we may see debt being used more often than equity. than the proprietorship, and riskier projects may or may not be more likely to be financed with either of the two organizational forms.

There is a large literature on the conflict between the manager and outside investors. Papers that are closely related to this paper are Townsend (1979), Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990), Hart and Moore (1998) and DeMarzo and Fishman (2000), which consider financing with outside debt, Myers (2000), which considers financing with outside equity, and Fluck (1998), which compares the two. With a few exceptions, these papers consider financing of projects whose cash flows are not publicly observable.

Fluck (1998) compares debt with equity, and finds that both debt and equity can be used to finance projects with infinite life, and that equity (weakly) dominates debt. An interesting idea of the paper is that giving equityholders the right to fire the manager whenever they want to can be an effective way to support outside equity. While the ideas in Fluck (1998) are interesting, the model is rather complicated and the analysis is hard to follow. The main result of the paper that equity dominates debt seems to be driven by an assumption on bankruptcy law, which allows the ownership of the project to be transferred to equityholders but not debtholders (even in bankruptcy). The (assumed) difference in debtholders’ and equityholders’ ability to replace the (current) manager (with a new manager) makes debt inferior. In this paper, we show that debt and equity induce different incentive problems, and that depending on the relative severity of the incentive problems, either debt or equity can dominate the other.

Our paper is related most closely to Myers (2000), a paper designed to illustrate Fluck’s ideas in a simple model. Myers (2000) considers equity financing in two organizational forms: partnership and corporation, and finds that while the partnership form can be used for both projects with infinite life and projects with finite life, the corporation form can be used only for projects with infinite life. It also finds that equity financing in both organizational forms requires co-investment by the manager. An important contribution of the paper is that it develops an elegant non-stochastic model for studying incentives. This paper compares a different pair of organizational forms, and describes when each or-

\[^{1}\text{Effort intensive firms and capital intensive firms here refer to respectively firms whose production requires a lot of managers’ effort and firms whose production requires a large initial investment.}\]
ganizational form dominates the other. We further extend the results in Myers (2000) by showing that reputation effect, together with other institutions such as employee share ownership and non-vested pensions, is sufficient to support equity financing, and that co-investment by the manager is not necessary. An important difference between the equity contract in this paper and the one in Fluck (1998) and Myers (2000) is that the equity contract gives the control over the firm’s output to equityholders in this paper but to the manager in the other two papers.

Bolton and Scharfstein (1990) and Hart and Moore (1998) consider financing of a project with finite life, and find that debt, but not equity, can be used to finance the project, and that inevitable early liquidation of the project can create certain inefficiency in debt financing. We show in this paper that with sinking funds, employee share ownership and non-vested pensions, both projects with infinite life and projects with finite life can be financed with both debt and equity, and that inefficient liquidation of the project can be avoided.

Townsend (1979), Diamond (1984) and Gale and Hellwig (1985) consider financing of a one-period project whose cash flow can be verified at a cost, and show that outside debt is the optimal contract. DeMarzo and Fishman (2000) considers financing of a project whose cash flows are nonobservable even to investors, and also finds that debt is the optimal contract.

The recent literature has made clear that the role of various contracts is much different in intertemporal settings than in the textbook one-period models. We choose to work with an intertemporal model because such a model makes it easier for us to study the incentives under reputation effect than a multi-period model. Like Myers (2000), we also look at incentives without any randomness. This means we cannot look at the asset substitution problem (i.e., the problem associated with the perverse incentive to substitute into a higher risk project to transfer value from bondholders to shareholders) that is often analyzed in single-period models, but we can still look at an interesting set of incentive issues in a tractable intertemporal model. An important difference between our model and the one in Myers (2000) is that the output depends on the manager’s effort in our model, but not in Myers’. The model in this paper is designed to highlight the differences between the corporate form and the proprietorship and to keep our analysis reasonably simple. We rule out the frictions that can affect debt and equity, such as taxes or issuing costs.

The rest of the paper is organized as follows. Section 2 introduces the model in the paper, and considers equity financing of the project. Section 3 considers debt financing of the project. Section 4 compares debt with equity. Our analysis in Section 2 through Section 4 is based on the assumptions that the threat of
dismissal and the threat of liquidation always exist and that the MBO is not possible. Section 5 considers the impact of relaxing these two assumptions on the financing of the project, and provides the intuitive reasoning for why the possibility of an MBO may jeopardize funding of the project and why the three institutions mentioned above can support the financing of the project. Section 6 formally defines the manager’s and investor’ choice problems and proves the results (stated) in Section 5. Section 7 concludes the paper.

2 The Model

This paper compares two different organizational forms for a profitable project whose cash flows are observable to both the manager and the investor, but not publicly verifiable. For illustrative purposes, we will tell the economic story of the paper by considering financing of a farm. There is nothing special about the farm upon which we have to rely to derive our results. Many firms, especially small firms, have the characteristics that are important in our model.

We assume that a manager, who is also the entrepreneur who owns the project idea initially, is capable of operating a farm. The cash flows from the farm are as follows. Initial investment in the land occurs at time 0 and costs an amount $K$. If the initial investment is made, the farm is in operation starting in year 1 through some date $T_{\text{sold}}$ (perhaps $T_{\text{sold}} = \infty$ indicating no sale) when the land is sold for $K$. The manager has a unique talent for making the plants grow quickly by talking to the plants, but cannot communicate this knowledge. During each year $t$ of operation, the farm generates a net cash flow (“income”)

$$ (1) \quad y_t = rK + \alpha e_t, $$

where $r$ is the riskfree rate, $\alpha > 1$ is a constant, and $e_t \in [0, \pi]$ is the manager’s effort (talking to the plants) at time $t$. We interpret $\alpha$ to be the manager’s efficiency. Absent effort, the farm earns a normal return $rK$ and the net present value of the project is zero. Since $\alpha > 1$, the first-best has the manager undertaking the maximum effort $\pi$ in all periods. If the manager leaves the farm for a year, the speech that makes the plants grow fast is forgotten. In this case, effort afterwards will forever be 0 and the farm may as well be sold since it will never earn more than the riskfree rate. We assume that the manager earns a normalized zero wage working outside the farm and that all the financial transactions between the manager and the investors can be verified at zero cost, but that contractual terms depending on nonpublic information rely on the agents’ incentive and cannot be enforced directly in court.
The manager’s utility function is $U^M(C^M_t, E_t) = \sum_{t=1}^{\infty} \frac{u(c^M_t, e_t)}{(1+r)^t}$, where the pure rate of time discount equals the riskfree rate $r$, the felicity function $u$ is

$$u(c, e) = \begin{cases} 
  c - e & \text{for } c \geq 0 \\
  -\infty & \text{otherwise} 
\end{cases},$$

and we use the conventions that $C^M_t \equiv \{c^M_t, c^M_{t+1}, \ldots\}$ is the manager’s consumption from time $t$ onwards and $E_t \equiv \{e_t, e_{t+1}, \ldots\}$ is the effort profile from time $t$ onwards. The manager has no endowment and therefore must turn to an outside investor to finance the project.

Outside investors are truly risk-neutral (unlike the manager who is risk-neutral for nonnegative consumption and infinitely risk averse for negative consumption). They care only about the expected net present value of any investment. Our formal model will assume there is a single (representative) investor, but assumptions we make in the model about bargaining power and sharing of rents will be motivated by competition among investors. The investor’s utility function is $U^I(C^I_0) = \sum_{t=0}^{\infty} \frac{c^I_t}{(1+r)^t}$, where, as for the entrepreneur, we define the investor’s consumption from $t$ onwards by $C^I_t$. The investor’s endowment (of consumption) is normalized to 0. The actual endowment is irrelevant; global linearity implies that the investor’s actual endowment does not affect the feasibility of net trades or preferences over them.

What makes contracting difficult is that neither effort nor income is publicly verifiable and therefore a contract depending on either cannot be enforced in court. However, it is possible to sell a share allowing the investor to go in and harvest the crop on the land; it is assumed that this can be done costlessly. In this case, the investor observes the size of the crop directly, and the manager knows it implicitly because the manager knows the level of effort. However, an outside court cannot verify what is common knowledge to the manager and the investor. Together with control rights to fire the manager and sell off the firm, this will constitute an equity claim. General nonlinear claims are not available in the model, because the output is observed only by the person who harvests it.

**Corporate Form**

We will assume that the manager-entrepreneur who sets up the firm captures all the rents. In the corporate form, the rents are captured by selling the land at a fair price. At the outset (time 0), the investor pays $K$ to buy equity in the
firm and the manager uses the proceeds to buy the parcel of land. In this case, the investor is the equityholder of the firm, and the entrepreneur works as an employee (manager) of the firm. In the model in this section, we do not allow MBOs.

The game between the employee-manager and the investor is a game of perfect information with sequential moves. In each period $t \geq 1$, we have the following stages:

1. The manager chooses effort $e_t \in [0, \bar{e}]$ (how long to talk to the plants).

2. The crop grows to produce output $y_t = rK + \alpha e_t$.

3. The investor harvests the entire crop and pays the manager the pre-agreed wage $w_t$ which is consumed by the manager $c_t = w_t$. (Later we will allow for saving by the manager, but for now saving by the manager would be redundant.) We will consider a candidate equilibrium in which $w_t = \alpha \bar{e}$, the manager’s marginal product assuming maximal effort. We assume that the equityholder is obligated to pay the manager the fixed wage (unlimited liability), and that the equityholder always has the ability to do so (abundant wealth).\footnote{The assumptions of unlimited liability and abundant wealth are made here to keep this part of the analysis simple. It can be shown that when $\alpha \bar{e} \leq K$, limited liability would give us the same result.} We will replace these two assumptions with the assumption of limited liability in our analysis in Section 6.

4. The investor chooses whether to liquidate: the indicator $l_t = 1$ if the investor chooses to liquidate the firm, fire the manager and sell the land for the amount $K$, while $l_t = 0$ if the investor retains the manager to operate the farm for another period. In our candidate equilibrium, the manager will be fired if less than maximal effort is exerted.

At each stage, the decision can depend on the entire history known to the agent. When the manager chooses $e_t$, that can depend on the entire previous history $h^t \equiv (E_{1:t-1}, L_{1:t-1})$, where $E_{1:t-1} \equiv (e_1, e_2, ..., e_{t-1})$ and $L_{1:t-1} \equiv (l_1, l_2, ..., l_{t-1})$, while the investor’s choice $l_t$ to sell the land and fire the manager depends on this history and this period’s effort. In other words, $e_t$ is chosen as a function of $h^t$ and $l_t$ is a function of $h^t$ and $e_t$.

The strategies for the two agents interact to create a trajectory through the game. We will describe this more formally in Section 6. Here, we state the choice problems at a time when the equity has already been issued and
the proceeds ($K$) have been used to purchase the parcel of land. The decision problem faced by the manager is:

**Problem 1 Employee-manager’s choice (no MBO)** Given the investor’s strategy for dismissal \( \{l_t(h^t, e_t)\} \), the employee-manager chooses the strategy for effort \( \{e_t(h^t)\} \) to maximize the expected lifetime utility of consumption and effort \( \sum_{t=1}^{\infty} \frac{c_t - e_t}{(1+r)^t} \) on the resultant trajectory, subject to \((\forall t) 0 \leq e_t \leq \bar{c} \) and \((\forall t) c_t = \alpha \bar{c} \Pi_{s=1}^{t-1} (1 - l_s)\). \(^3\)

The decision problem to be faced by the investor is (again leaving the more formal description to Section 6):

**Problem 2 Investor’s choice (no MBO)** Given the employee-manager’s strategy for effort \( \{e_t(h^t)\} \), the investor chooses the strategy for dismissal \( \{l_t(h^t, e_t)\} \) to maximize the expected lifetime cash flow \(-K + \sum_{t=1}^{\infty} \frac{(rK + \alpha e_t - \bar{c}) \Pi_{s=1}^{t-1} (1 - l_s)}{(1+r)^t} \) along the resultant trajectory, with the usual convention that an empty product is defined to be equal to 1. \(^4\)

Our equilibrium concept is subgame perfect Nash equilibrium, so there will be a similar problem to be solved at each date and for every possible history, and not just for the histories that would be realized along the equilibrium path. This standard equilibrium concept seems like a reasonable one in this model (what further justification do we have for different equilibrium concepts?) and puts reasonable discipline on expectations about how the other player will continue if one player deviates from the candidate optimal strategy. Here is our main result describing the corporate equilibrium.

**Theorem 1 Equilibrium (Outside Equity)** In the simple corporate model without MBOs, the first-best has effort \( e_t \) equal to its maximum value \( \bar{c} \) along the equilibrium path in every period (since we are assuming that the marginal product of effort \( \alpha \) is greater than the marginal cost 1). Whether this first-best can be achieved depends on how large is the effectiveness of effort \( \alpha \): (i) If \( \alpha \geq 1 + r \), then the first-best (with \( e_t = \bar{c} \) along the equilibrium path) can be implemented using equity,\(^5\) using the “good corporate equilibrium” below. (ii) If

\(^3\)The product \( \Pi_{s=1}^{t-1} (1 - l_s) \) is 0 if there has been a liquidation before \( t \) and 1 otherwise. We follow the usual convention that an empty product equals 1.

\(^4\)The denominator to \( K \) discounts from the liquidation date to date 0. If there is no liquidation, the liquidation date “is” \( \infty \) and we take the term to be zero.

\(^5\)Implementation means in our assumed equilibrium concept, subgame perfect Nash equilibrium in pure strategies.
\(\alpha < 1 + r\), then the first-best (with \(e_t = \tau\)) cannot be implemented, and in fact no equilibrium has positive effort along the equilibrium path. Good corporate equilibrium strategies are: the employee-manager always expends the first-best in equilibrium

\[
\begin{align*}
(2) \quad e_t(h^t) &= \begin{cases} 
\tau & \text{if } (\forall s < t) e_s = \tau \\
0 & \text{otherwise}
\end{cases},
\end{align*}
\]

and the investor fires the manager as punishment for less than the first-best

\[
\begin{align*}
(3) \quad l_t(h^t, e_t) &= \begin{cases} 
0 & \text{when } (\forall s \leq t) e_s = \tau \\
1 & \text{otherwise}
\end{cases}.
\end{align*}
\]

**Proof:** Appendix A.

The agents' strategies in the good corporate equilibrium consist of two parts: the good strategies (played on the equilibrium path) and the bad or punishment strategies (played on every off-equilibrium path). The first-best effort in the equilibrium is supported by the threat to play the punishment strategies forever once one party deviates from the good strategy, and it can be supported only when \(\alpha \geq 1 + r\), i.e., when the effectiveness of effort is large enough. As is normal in Nash Equilibrium, the off-equilibrium strategy of an agent is important because it describes expectations about what would happen after a deviation. Assuming no effort after a deviation makes credible the extreme punishment that might be needed to support equilibrium.

### 3 Proprietorship

Another way to finance the project is to use debt. At the outset (time 0), the manager borrows \(K\) from an investor to buy the parcel of land. In return, the manager promises the investor an interest payment in every subsequent period. In this case, the manager is also the equityholder of the firm, and the investor is the debtholder of the firm. In the model in this section, we do not allow the manager to take over the firm by paying off all the outstanding debt (i.e. to take over the firm through debt-repurchase).

The game between the owner-manager and the investor is a game of perfect
information with sequential moves. In each period \( t \geq 1 \), we have the following stages:

1. The manager chooses effort \( e_t \in [0, \overline{e}] \). It is always optimal to choose the maximal effort level: \( e_t = \overline{e} \).

2. The crop grows to produce output \( y_t = rK + \alpha e_t \).

3. The owner-manager harvests the entire crop, and then chooses the interest payment \( i_t \) to the investor and consumers all the income in the period (i.e. \( c_t = y_t - i_t \)). (We will allow for saving by the manager in Section 6).

4. The investor needs to decide whether to force the firm into bankruptcy when a promised interest payment is missed: indicator \( l_t = 1 \) if the investor chooses to force the firm into bankruptcy, which leads to the firing of the manager, liquidation of the firm and sale of the land by our assumption of no debt-repurchase, while \( l_t = 0 \) if the investor decides to give the manager the control of the farm for another period.

5. In case the firm is forced into the bankruptcy, the land will be sold in a second-price auction, proceeds will be used to pay off the debt, and the remainder (if any) will go to the owner-manager.

Same as in the game in equity financing, an agent’s decision at each stage can depend on the entire history known to the agent. In this game, the manager’s choice \( i_t \) depends on \( h^t \equiv (I_{1,t-1}, L_{1,t-1}) \), where \( I_{1,t-1} \equiv (i_1, i_2, ..., i_{t-1}) \) and \( L_{1,t-1} \equiv (l_1, l_2, ..., l_{t-1}) \), and the investor’s choice \( l_t \) depends \( h^t \) and \( i_t \).

The strategies for the two agents interact to create a trajectory through the game. Here, we state the agents’ choice problems at a time when the money has already been borrowed and the purchase of the parcel of land has already been done. The decision problems to be faced by the manager and the investor are:

**Problem 3 Owner-manager’s choice (no Debt-Repurchase)** Given the investor’s strategy for liquidation \( \{i_t(h^t, i_t)\} \), the owner-manager chooses the strategy for interest payment \( \{i_t(h^t)\} \) to maximize the expected lifetime utility of consumption and effort \( \sum_{t=1}^{\infty} \frac{\alpha - r \prod_{s=1}^{t-1}(1-l_s)}{(1+r)^t} \) along the resultant trajectory, subject to \( (\forall t)c_t = (rK + \alpha e_t - i_t) \prod_{s=1}^{t-1}(1-l_s) \).

**Problem 4 Investor’s choice (no Debt-Repurchase)** Given the owner-manager’s strategy for interest payment \( \{i_t(h^t)\} \), the investor chooses the strat-
egy for liquidation \( \{l_t(h^t, i_t)\} \) to maximize the expected lifetime cash flow \(-K + \sum_{t=1}^{\infty} \frac{i_t \prod_{s=1}^{t-1}(1-l_s)}{(1+r)^t} \) along the resultant trajectory.

Here is our main result describing the proprietorship equilibrium.

**Theorem 2 Equilibrium (Debt)** In the simple proprietorship model without debt-repurchase, whether the interest payments over time \( \{i_t^* = rK, t \geq 1\} \) can be maintained depends on how large is the effectiveness of effort \( \alpha \): (i) If \( \alpha \geq 1 + \frac{r^2K}{e} \), then the interest payments can be maintained, using the “good proprietorship equilibrium” below. (ii) If \( \alpha < 1 + \frac{r^2K}{e} \), then interest payments cannot be maintained, and in fact no equilibrium has positive interest payments along the equilibrium path. Good proprietorship equilibrium strategies are: the owner-manager always makes the promised interest payment in equilibrium

\[
(4) \quad i_t(h^t) = \begin{cases}  
    i_t^* & \text{if } (\forall s < t)i_s \geq i_s^* \\
    0 & \text{otherwise}
\end{cases},
\]

and the investor forces the firm into bankruptcy as punishment for less than the promised interest payment

\[
(5) \quad l_t(h^t, i_t) = \begin{cases}  
    0 & \text{when } (\forall s \leq t)i_s \geq i_s^* \\
    1 & \text{otherwise}
\end{cases}.
\]

**Proof:** See Appendix A.

### 4 Corporate Form versus Proprietorship

Our analysis in the previous two sections shows that the two organizational forms can have very different incentive problems. In equity financing, the employee-manager signs a fixed-wage contract and works as an employee of the firm. Given that the manager’s income is guaranteed by the wage contract and does not depend on the output of the firm, the manager may have an incentive not to exert any effort on the project and just collect the wage. The incentive problem in the corporate form is that the employee-manager may have an incentive to shirk.
In debt financing, the owner-manager borrows from an investor and promises the investor an interest payment in each subsequent period. As the residual claimant on the firm’s cash flow and value, the manager has no incentive to shirk, but may have an incentive to take the entire cash flow out of the firm and default on the debt. The incentive problem in the proprietorship is that the owner-manager may have an incentive not to make any interest payment.

The analysis produces the following result on the feasibility of the two organizational forms:

**Theorem 3 Corporate Form versus Proprietorship** We have that (i) neither the corporate form nor the proprietorship can be used when \( \frac{(\alpha-1)\pi}{r} < \min\{rK, \tau\} \); (ii) the proprietorship, but not the corporate form, can be used when \( rK \leq \frac{(\alpha-1)\pi}{r} < \tau \); (iii) the corporate form, but not the proprietorship, can be used when \( \tau \leq \frac{(\alpha-1)\pi}{r} < rK \); and (iv) both the corporate form and the proprietorship can be used when \( \max\{rK, \tau\} \leq \frac{(\alpha-1)\pi}{r} \).

**Proof:** Immediate from Theorem 1 and Theorem 2.

It is worth explaining the intuition behind the result in the theorem. The term \( \frac{(\alpha-1)\pi}{r} \) equals the value of the firm to the manager. It also equals the loss that the manager will suffer when leaving the firm. The terms \( \tau \) and \( rK \) equal respectively the gain for the manager from shirking and the gain from not making the interest payment. The result in the theorem implies that the corporate form and the proprietorship each can be used only when the manager cannot benefit from shirking or defaulting on the debt, and that depending on the relative severity of the two incentive problem, either organizational form can dominate the other.

The model has the following predictions on the optimal choice between the two organizational forms:

1. The corporate form should be the preferred choice of the two when the production is not effort intensive, but capital intensive.
2. The proprietorship should be the preferred choice of the two when the production is not capital intensive, but effort intensive.

In our example of the farm, these predictions imply that the corporate form, not the proprietorship, should be used when it is not a lot of work for the manager to talk to the plants, but the initial investment on the farm is large relative.
to the future profit of the farm. The proprietorship, not the corporate form, should be used when it is a lot of work for the manager to talk to the plants, but the initial investment of the farm is small relative to the future profit of the farm.

5 Impact Of An Aftermarket: Preview of Results

Three assumptions were made to keep the analysis in the last three sections reasonably simple. The first assumption is that neither MBO nor debt-repurchase is possible. The second assumption is that the project has an infinite life. The third assumption is that the manager is risk-neutral and there is no uncertainty over the output of the project. In this section, we discuss how relaxing each of the three assumptions would change the results in the previous sections. We find that when the first two assumptions do not hold, the threat to fire the manager or to liquidate the project itself cannot prevent the manager from shirking or defaulting on the debt, and the financing of the project can be supported by institutions such as sinking funds, stock or option compensation and managers’ non-vested pensions. When the manager is risk-averse and when there is some uncertainty over the output of the project, an increase in the manager’s risk-aversion tends to increase the likelihood that equity is the preferred choice of the two, and an increase in the riskiness of the project can either increase or decrease this likelihood. We provide only the intuitive reasoning for these findings in this section, and we will formally prove (some of) these results in the next section.

Failure of Reputation Effect to Support Financing

Our analysis in the previous sections shows that to support the financing of the project, the investor has to be able to punish the manager for deviations from the good strategies in all periods. The investor will not be able to do so when the first two assumptions stated above do not hold. When the project or the manager has a finite life, the investor will not be able to punish the manager for shirking or defaulting on the debt in the last period of the project’s or the manager’s life. When an MBO or debt-repurchase is possible, firing the manager or liquidating the project is not a punishment to the manager anymore, because the manager can simply take over the firm and reinstate the project right after being fired. In both cases, besides the threat of dismissal and the threat of
liquidation, some institutions need to be used to support the financing of the project.

**Financing with Different Institutions**

We find that three institutions can be used to support the financing of the project: debt financing can be supported by sinking funds, and equity financing can be supported by stock or option compensation or managers’ non-vested pensions.\(^6\) In this subsection we provide the intuitive reasoning for this result.

**Financing with Sinking Funds**

When debt is used, the lender can require the borrower to retire a (pre-specified) fraction of the (principal of the) debt periodically through a sinking fund.\(^7\) If we design the sinking fund of a debt in such a way that a large percentage of the debt is retired early, then the debt can be used to finance the project. The intuition is that if a large enough proportion of the debt is retired early enough, there won’t be any default on the debt because the payment of the remaining debt can be guaranteed by the liquidation value of the project. The quickest way to retire all the debt is to ask the manager to spend all the income on debt-repurchase.

**Financing with Non-Vested Pensions**

Each employee’s pension can have a non-vesting period. An employee may lose the pension if leaving the firm during this period. Managers’ non-vested pensions can be used to support equity financing of the project. If we design the manager’s pension in such a way that the manager loses the pension to a third party (such as the Pension Benefit Guarantee Corporation) when leaving the firm before the pension becomes vesting, the investor should be able to deter the manager’s shirking by holding the pension against the shirking. When the life of the project is longer than the non-vesting period of the pension, it may

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\(^6\) All stock compensation in the paper are assumed to be in the company’s non-voting shares. In our example of the farm, each share of the company’s stock is a claim on (a fraction of) the farm’s land. Since the value of the land is known to everyone, claims on the ownership of the land can always be traded publicly.

\(^7\) This provision in the debt contract is referred to as the sinking fund requirement.
be optimal to set up the manager’s pension in such a way that it covers the later part of the project’s life.

Financing with Stock or Option Compensation

Stock or option compensation can also be used to support equity financing of the project. Its ability to do so depends crucially on two important properties of this compensation. One property is that this compensation allows the complete transfer of the firm’s ownership from the investor to the manager. The other property is that it can eliminate the manager’s incentive to shirk: when the manager’s ownership of the firm is large enough, the manager has no incentive to shirk. The biggest challenge in using this compensation, which is also a basic requirement for being able to use this compensation, is to find a feasible way to divide the firm’s output between the two parties according to the fractions of their ownership of the firm in each period, when the output of the firm itself is not publicly verifiable.

In our analysis of equity financing in Section 2, the investor is given the control over the output of the firm. The wage payment to the manager is guaranteed by the liquidation value of the project (under the limited liability assumption). When the manager is compensated in the company’s shares, the control over the firm’s cash flows can no longer be given to any single party. Giving the control to the investor will create another incentive problem (which can be called the incentive problem for the investor): the investor may have an incentive not to make any dividend payment to the manager. It can be shown that it can be very difficult to deal with the two incentive problems and have them both under control at the same time.

One way to split the firm’s output between the two parties is to give the control over one part of the output to one party and the control over the other part of the output to the other party. This can be done by dividing the overall product market into many sub-markets and allowing each party to collect revenue from sales in different sub-markets (as dividends and possibly the wage). In our example of the farm, this is equivalent to letting each party harvest crops on different parts of the land. One way to ensure that the manager’s shirking reduces the manager’s dividend income but not the wage income is to pay the manager the wage in the company’s shares and the dividends in cash (collected from sales in certain sub-markets).

It is possible to pay the manager the wage and dividends in almost every possible combination of the company’s shares and cash. This can be done by
paying the manager the wage in the company’s shares and dividends in cash and then exchanging the shares for cash of equal value or vice versa.\(^8\)

The output-sharing between the manager and the investor can be both difficult and tedious to arrange at times.\(^9\) The potential difficulties and tediousness associated with arranging this output-sharing may prevent certain projects from being financed with equity, and may also make debt the preferred choice of the two when both can be used. As a result, we may see debt being used more often than equity.

**Financing of Projects with Finite Life**

All three institutions discussed above can be used to support financing of projects with finite life. The intuition behind this result is that use of sinking funds makes it possible to retire all the debt before the final period of the project’s life and therefore avoid default on debt in that period. Use of stock or option compensation makes it possible to complete the transfer of the firm’s ownership from the investor to the manager before the final period of the project’s life and therefore avoid the manager’s shirking in that period. Use of non-vested pensions makes it possible for the investor to punish the manager for shirking in the last period of the project’s life.

**Effects of a Change in Manager’s Risk-Aversion and Project’s Riskiness on Financing**

When the manager is risk-averse and when there is some uncertainty over the output of the project, a change in the manager’s risk-aversion or the riskiness of the project can lead to a change in the optimal choice between the two organizational forms. Because the manager bears more risk with debt than with equity, an increase (decrease) in the manager’s risk-aversion tends to increase the likelihood that the corporate form (proprietorship) would be the preferred choice of the two. An increase in the riskiness of the project can increase the likelihood that either of the two organizational forms is the preferred choice of the two. On the one hand, an increase in the riskiness of the project is likely to reduce the manager’s expected utility more with the proprietorship than with the corporate form, because such an increase increases the volatility of the

\(^8\)This exchange should be very easy because the two are of equal value.

\(^9\)One requirement that needs to be satisfied for this output-sharing is the investor’s direct involvement in the operation of the firm.
manager's total income more with the proprietorship than with the corporate form. On the other hand, an increase in the riskiness of the project can make it more difficult for the investor to infer the manager’s effort from the output of the firm and therefore makes it more difficult for the corporate form to be used. Depending on the relative magnitude of the two opposite effects, either organizational form can be optimal.

6 Impact Of An Aftermarket: Formal Models

In this section, we formally prove some of the results stated in Section 5. We show that when the manager is allowed to take over the firm, the threat of dismissal or the threat of liquidation itself cannot prevent the manager from shirking or defaulting on the debt, and the financing of the project can be supported by sinking funds, stock or option compensation and managers' non-vested pensions. First we consider equity financing. Then we consider debt financing.

The agents’ choice problems in this section are much more complicated than the ones in Section 2 and Section 3 mainly because without the restrictions on MBO and debt-repurchase, the manager needs to make a few more decisions in each period, and as a result, we need to keep track of the values of many more variables and consider all possible outcomes of the agents’ actions.

Agents’ Choice Problems in Equity Financing

In this subsection we set up the agents’ choice problems in equity financing for a project with an infinite life. First, we define the variables

\( e_t \) — manager’s effort in period \( t \): \( e_t \in [0, \bar{e}] \).

\( w_t \) — manager’s wage in period \( t \): \( w_t \geq 0 \).

\( d_t \) — manager’s dividend income in period \( t \): \( d_t \geq 0 \).

\( c_t \) — manager’s consumption in period \( t \): \( c_t \geq 0 \).

\( \Delta \beta_{w,t} \) — fraction of the ownership of the company that the manager receives as part of the wage income in period \( t \) (which is a choice variable for the investor): \( \Delta \beta_{w,t} \in [0, 1] \).

\( \Delta \beta_{d,t} \) — fraction of the ownership of the company that the manager receives as part of the dividend income in period \( t \) (which is a choice variable for the
investor): $\Delta \beta_{d,t} \in [0, 1].^{10}$

$\beta_t$—fraction of the company owned by the manager at the end of period $t$: $\beta_t \in [0, 1]$ and $\beta_t \equiv \beta_{t-1} + \Delta \beta_{w,t} + \Delta \beta_{d,t}$.

$j_t$—manager’s total income from the project in period $t$, which includes both the manager’s wage income and dividend income.

$m_t$—cash balance in the manager’s saving account (or money market account) at the beginning of period $t$.

$W_t$—manager’s total wealth at the end of period $t$, which includes the manager’s ownership of the firm.

$y_t$—output of the project in period $t$.

$l_t$—investor’s decision on firing the manager in period $t$: $l_t = 1$ if the manager is fired in period $t$ and $l_t = 0$ not.

Next, we define the agents’ strategies:

$\sigma_{m,e}$—manager’s strategy on effort: $\sigma_{m,e} : h^t \mapsto [0, \bar{e}]$ and $e_t \equiv \sigma_{m,e}(h^t)$.

$\sigma_{m,c}$—manager’s strategy on consumption: $\sigma_{m,c} : h^t \mapsto [0, W_t]$ and $c_t \equiv \sigma_{m,c}(h^t)$.

$\sigma_{i,l}$—investor’s strategy on firing the manager: $\sigma_{i,l} : (h^t, e_t) \mapsto \{0, 1\}$ and $l_t \equiv \sigma_{i,l}(h^t, e_t)$.

$\sigma_{\Delta b,w}$—investor’s strategy on paying part or all the manager’s wage income in the company’s shares instead of cash: $\sigma_{\Delta b,w} : (h^t, e_t) \mapsto [0, 1]$ and $\Delta \beta_{w,t} \equiv \sigma_{\Delta b,w}(h^t, e_t)$.

$\sigma_{\Delta b,d}$—investor’s strategy on paying part or all the manager’s dividend income in the company’s shares instead of cash: $\sigma_{\Delta b,d} : (h^t, e_t) \mapsto [0, 1]$ and $\Delta \beta_{d,t} \equiv \sigma_{\Delta b,d}(h^t, e_t)$.

$\sigma_m \equiv (\sigma_{m,e}, \sigma_{m,c})$ and $\sigma_i \equiv (\sigma_{\Delta b,w}, \sigma_{\Delta b,d}, \sigma_{i,l})$.

Each pair of the agents’ strategies creates a trajectory of the outcomes of the game between the two agents. Now we define the trajectory of the outcomes determined by $\sigma_m$ and $\sigma_i$. For each triple $(h^t, \sigma_m, \sigma_i)$, we define trajectory$^{11}$

$$h_t^\infty = \{(e_1, e_2, ...), (\Delta \beta_{w,1}, \Delta \beta_{w,2}, ...), (\Delta \beta_{d,1}, \Delta \beta_{d,2}, ...), (l_1, l_2, ...)\}$$

recursively:

$$e_s = \begin{cases} e_s & \text{for } s < t \\ \sigma_{m,e} ((E_{1,s-1}), (\Delta \beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}$$

$^{10}$ $\Delta \beta_{w,t}K$ and $\Delta \beta_{d,t}K$ equal respectively the values of the parts of the wage and dividend income that the manager receives in the company’s shares.

$^{11}$ An implicit assumption made here is that the manager’s consumption is not observable to others and therefore cannot be used by the equityholder in the decision-making. In case the manager’s consumption is observable to the equityholder, the financing of the project can be supported by requiring that the manager consumes all or most of the income in each period.
\[ c_s = \begin{cases} c_s & \text{for } s < t \\ \sigma_{m,c}(E_{1,s-1}, (\Delta \beta_{1,s-1})(L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ \Delta \beta_{w,s} = \begin{cases} \Delta \beta_{w,s} & \text{for } s < t \\ \sigma_{A_{b,w}}(E_{1,s}, (\Delta \beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ \Delta \beta_{d,s} = \begin{cases} \Delta \beta_{d,s} & \text{for } s < t \\ \sigma_{A_{b,d}}(E_{1,s}, (\Delta \beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ l_s = \begin{cases} l_s & \text{for } s < t \\ \sigma_{i,t}(E_{1,s}, (\Delta \beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases} \]

\[ y_s = \begin{cases} y_s & \text{for } s < t \\ rK + \alpha e_s & \text{otherwise} \end{cases} \]

where \( E_{1,t-1} \equiv (e_1, ..., e_{t-1}) \), \( \Delta \beta_{1,t-1} \equiv (\Delta \beta_{w,1,t-1}, \Delta \beta_{d,1,t-1}) \), \( \Delta \beta_{w,1,t-1} \equiv (\Delta \beta_{w,1}, ..., \Delta \beta_{w,t-1}) \), \( \Delta \beta_{d,1,t-1} \equiv (\Delta \beta_{d,1}, ..., \Delta \beta_{d,t-1}) \), and \( L_{1,t-1} \equiv (l_1, l_2, ..., l_{t-1}) \).

The time when the manager is fired and the project is sold along the trajectory is given by \( T_s \equiv \inf \{t | l_t = 1 \text{ and } W_t < K, t \geq 1\} \), and the time when an MBO occurs along the trajectory is given by \( T_m \equiv \inf \{t | l_t = 1 \text{ and } W_t \geq K, t \geq 1\} \).

For the convenience of our discussion, now we define three sets of constraints: one on the manager’s strategy, one on the investor’s strategy, and one a set of variables. The constraints on the manager’s strategy, which specify the range and initial values of the manager’s choice variables, are

\[ c_t \in [0, W_t], \quad e_t \in [0, \bar{e}], \quad m_t \geq 0, \quad m_0 \equiv 0. \]

The constraints on the investor’s strategy, which specify the range and (initial) values of the investor’s choice variables, are

\[ (\Delta \beta_{w,t}, \Delta \beta_{d,t}, \beta_t) \in [0, 1]^3, \]
\[ (\Delta \beta_{0}, \Delta \beta_{d,0}, \beta_0) \equiv 0, \]

\[ \Delta \beta_{w,t} \in \left[ \frac{\alpha \bar{e} K}{r}, \frac{\alpha \bar{e}}{K} \right] \text{ when } t < \min \{T_m, T_s\}, \]
\[ \Delta \beta_{w,t} = 0 \text{ when } t \geq \min \{T_m, T_s\}, \]
\[ \Delta \beta_{d,t} \in \left[ 0, \frac{d_t}{K} \right] \text{ when } t < \min \{T_m, T_s\}, \]
\[ \Delta \beta_{d,t} = 0 \text{ when } t \geq \min \{T_m, T_s\}, \]

\[ \beta_t \equiv \beta_{t-1} + \Delta \beta_{w,t} + \Delta \beta_{d,t}. \]

\footnote{It is optimal for the manager to take over the firm before it is sold to others. Otherwise, the manager would forget the speech that can make the plants grow.}
The lower bound of $\Delta \beta_{w,t}$ (which equals $\frac{\alpha e}{K} - r$) in (7) guarantees the non-negativity of the firm’s dividend payment.

The constraints on the following set of variables: firm’s output, the manager’s wage, the manager’s dividend income, the manager’s total income from the project, cash balance in the manager’s saving account, and the manager’s total wealth, are

\begin{align*}
y_t &= rK + \alpha e_t, \\
w_t &= \begin{cases} 
w^*_t & \text{when } t \leq \min\{T_S, T_M\} \\
0 & \text{when } t > \min\{T_S, T_M\} \\
\end{cases}, \\
d_t &= \begin{cases} 
\beta_{t-1}(y_t - (w^*_t - \Delta \beta_{w,t}K)) & \text{when } t \leq \min\{T_S, T_M\} \\
0 & \text{when } t > \min\{T_S, T_M\} \\
\end{cases}, \\
\dot{j}_t &= \begin{cases} 
w_t + d_t & \text{when } t \leq \min\{T_S, T_M\} \\
y_t & \text{when } t > T_M \text{ and } T_M < T_S \\
0 & \text{when } t > T_S \text{ and } T_S < T_M \\
\end{cases}, \\
m_{t+1} &= \begin{cases} 
m_t + j_t - (\beta_t - \beta_{t-1})K - c_t & \text{when } t < \min\{T_M, T_S\} \\
m_t + j_t - (1 - \beta_t)K - c_t & \text{when } t = T_M \text{ and } T_M < T_S \\
m_t + j_t - c_t & \text{when } t > T_M \text{ and } T_M < T_S \\
m_t + j_t + \beta_{t-1}K - c_t & \text{when } t = T_S \text{ and } T_S < T_M \\
m_t - c_t & \text{when } t > T_S \text{ and } T_S < T_M \\
\end{cases}, \\
W_t &= m_t + j_t + \beta_{t-1}K,
\end{align*}

where $w^*_t = \alpha e$. The manager’s dividend income ($d_t$) in (8) is determined based on the assumption that the manager’s wage has to be paid in full before any dividend payment can be made by the firm. The fifth equality in (8) provides the cash balance of the manager’s saving account under different circumstances.

Now we define the agents’ choice problems at a time when the equity has already been issued and the parcel of land has already been purchased. The decision problems to be faced by the manager and investor are

**Problem 5 Manager’s choice (MBO)** Given the wage contract \( \{w^*_t = \alpha e, t \geq 1\} \) and the investor’s strategy satisfying the constraints in (7), the manager chooses $\sigma_m$ to maximize the expected lifetime utility of consumption and effort $\sum_{t=1}^{+\infty} \frac{c_t - \dot{e}_t}{(1+r)^t}$ subject to the constraints in (6) and (8).

**Problem 6 Investor’s choice (MBO)** Given the wage contract \( \{w^*_t = \alpha e, t \geq 1\} \) and the manager’s strategy $\sigma_m$ satisfying the constraints in (6), the investor
chooses \( \sigma_i \) to maximize the expected lifetime cash flow \( \sum_{t=1}^{T} e_t y_t - w_t^* - d_t + (\beta_t - \beta_{t-1})K \) subject to the constraints in (7) and (8), where \( T_e \equiv \min\{T_s, T_m\} \).

**Results on Equity Financing**

We have the following result on equity financing of the project.

**Theorem 4 Equilibrium (Outside Equity with MBO)** In the simple corporate model with MBOs, the first-best has effort \( e_t \) equal to its maximum value \( \overline{e} \) along the equilibrium path in every period. Whether this first-best can be achieved depends on both how large is the effectiveness of effort \( \alpha \) and whether employee share ownership or manager’s non-vested pension is used:

(i) **Financing cannot be supported by the threat of dismissal itself:** When neither the employee share ownership nor the manager’s non-vested pension is used, the first-best cannot be implemented, and in fact no equilibrium has positive effort along the equilibrium path.

(ii) **Financing can be supported by employee share ownership:** With employee share ownership, the first-best can be implemented using the “good corporate equilibrium” below when \( 1 + r \leq \alpha < \frac{K}{\overline{e}} \) and \( \beta_{T_{mbo}} \geq \frac{1}{\alpha} \), where \( T_{mbo} \equiv \inf\{t|\frac{\alpha e((1+r)^t-1)}{r} \geq K\} \) and \( \beta_{T_{mbo}} = \frac{\alpha e((1+r)^t_{mbo}-1)}{rK} \), provided the investor and the manager can find a feasible way to divide the firm’s output between them according to the fractions of their ownership of the firm in each period. The good corporate equilibrium that provides the quickest transfer of the firm’s ownership from the equityholder to the manager is the equilibrium with zero consumption and compensation in only the company’s shares:

\[
\begin{align*}
e_t(h^t) &= \begin{cases} \overline{e} & \text{if } (\forall s < t) e_s = \overline{e} \\ 0 & \text{otherwise} \end{cases}, \\
c_t(h^t) &= 0 \text{ when } \beta_{t-1} < 1, \\
l_t(h^t, e_t) &= \begin{cases} 0 & \text{when } (\forall s \leq t) e_s = \overline{e} \\ 1 & \text{otherwise} \end{cases}, \\
\beta_t(h^t, e_t) &= \min\{\beta_{t-1}(1+r) + \frac{\alpha e}{K}, 1\} \text{ when } \beta_{t-1} < 1.
\end{align*}
\]

(iii) **Financing can be supported by non-vested pension:** If the manager’s pension is designed in such a way that its non-vesting period is as long as the life of the project (which is infinity) and that the manager loses the pension to a third party when leaving the firm, then the first-best can be implemented for all \( \alpha > 0 \).
Now we provide an intuitive explanation of some of the results in the theorem. To support equity financing with employee share ownership, two conditions need to be satisfied. One condition is that the project can neither be too profitable nor be too unprofitable. The manager is always better off shirking if the value of the project is smaller than the costly effort to the manager (i.e. $(\alpha - 1)F < \tau$). The manager is always better off shirking and then taking over the project if the manager’s wage is larger than the initial investment of the project (i.e. $\alpha F \geq K$). The other condition is that the manager needs to own a large enough fraction of the firm by the time taking over the firm is possible. The reason is that the manager has no incentive to shirk only when the manager’s ownership of the firm is large enough. To support equity financing with manager’s non-vested pension, the pension needs to have a long enough non-vesting period.

Agents’ Choice Problems in Debt Financing

In this subsection, we set up the agents’ choice problems in debt financing for a project with an infinite life. First, we define the (additional) variables that are needed for the analysis in this subsection:

- $i_t$—interest payment in period $t$: $i_t \in [0, y_t]$.
- $\beta_t$—fraction of the company’s total debt retired by the sinking fund at the end of period $t$ (which is a choice variable for the manager): $\beta_t \in [0, 1]$.
- $l_t$—investor’s decision on forcing the firm into bankruptcy in period $t$: $l_t = 1$ if force the firm into bankruptcy in period $t$ and $l_t = 0$ if not.

Next, we define the agents’ strategies:

- $\sigma_{m,i}$—manager’s strategy on interest payment: $\sigma_{m,i} : h_t : [0, y_t] \mapsto i_t$ and $i_t \equiv \sigma_{m,i}(h_t)$.
- $\sigma_{m,c}$—manager’s strategy on consumption: $\sigma_{m,c} : h_t : [0, W_t] \mapsto c_t$ and $c_t \equiv \sigma_{m,c}(h_t)$.
- $\sigma_{m,b}$—manager’s strategy on retiring part or all the outstanding debt with the sinking fund: $\sigma_{m,b} : (h_t) : [0, 1] \mapsto \beta_t$ and $\beta_t \equiv \sigma_{m,b}(h_t)$.
- $\sigma_{i,t}$—investor’s strategy on forcing bankruptcy: $\sigma_{i,t} : (h_t, e_t, p_t) : \{0, 1\} \mapsto l_t$ and $l_t \equiv \sigma_{i,t}(h_t, e_t, p_t)$.
- $\sigma_m \equiv (\sigma_{m,i}, \sigma_{m,c}, \sigma_{m,b})$ and $\sigma_i \equiv \sigma_{i,t}$.

Each pair of the agents’ strategies creates a trajectory of the outcomes of the game between the two agents. Now we define the trajectory of the outcomes.
determined by $\sigma_m$ and $\sigma_i$. For each triple $(h', \sigma_m, \sigma_i)$, we define trajectory $h^\infty = \{(i_1, i_2, \ldots), (\beta_1, \beta_2, \ldots), (l_1, l_2, \ldots)\}$ recursively:

- $i_s = \begin{cases} i_s & \text{for } s < t \\ \sigma_{m,i} ((I_{1,s-1}), (\beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}$
- $c_s = \begin{cases} c_s & \text{for } s < t \\ \sigma_{m,c} ((I_{1,s-1}), (\beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}$
- $\beta_s = \begin{cases} \beta_s & \text{for } s < t \\ \sigma_{m,b} ((I_{1,s-1}), (\beta_{1,s-1}), (L_{1,s-1})) & \text{otherwise} \end{cases}$
- $l_s = \begin{cases} l_s & \text{for } s < t \\ \sigma_{i,l} ((I_{1,s}), (\beta_{1,s}), (L_{1,s-1})) & \text{otherwise} \end{cases}$
- $y_s = \begin{cases} y_s & \text{for } s < t \\ rK + \alpha \bar{e} & \text{otherwise} \end{cases}$

where $I_{1,t-1} \equiv (i_1, \ldots, i_{t-1})$, $\beta_{1,t-1} \equiv (\beta_1, \ldots, \beta_{t-1})$, and $L_{1,t-1} \equiv (l_1, \ldots, l_{t-1})$. The time when the manager is fired and the firm is forced into bankruptcy along the trajectory is given by $T_b \equiv \inf \{t| l_t = 1 \text{ and } W_t < (1+r)(1-\beta_{t-1})K, t \geq 1\}$, and the time when a take-over through debt-repurchase occurs along the trajectory is given by $T_t \equiv \inf \{t| l_t = 1 \text{ and } W_t \geq (1+r)(1-\beta_{t-1})K, t \geq 1\}$.

For the convenience of our discussion, now we define two sets of constraints, one on the manager’s strategy and the other on a set of variables. The constraints on the manager’s strategy, which specify the range and values of the manager’s choice variables, are

\begin{equation}
(10) \quad e_t \in [0, \bar{e}], \quad c_t \in [0, W_t], \quad i_t \in [0, y_t], \quad \beta_t \in [0, 1],
\end{equation}

- $m_t \geq 0$, $\beta_0 = 0$, $m_0 \equiv 0$, $e_t = \begin{cases} \bar{e} & \text{when } t \leq T_b \\ 0 & \text{when } t > T_b \end{cases}$,
- $\beta_t = \begin{cases} 1 & \text{when } t \geq T_t \text{ and } T_t < T_b \\ 0 & \text{when } t \geq T_b \text{ and } T_b < T_t \end{cases}$.

The constraints on the following set of variables: firm’s output, the manager’s total income from the project, the cash balance in the manager’s saving
account, and the manager’s total wealth, are

\begin{equation}
(11) \quad \begin{align*}
y_t &= rK + \alpha e_t, \\
j_t &= \begin{cases} 
y_t - i_t & \text{when } t \leq \min\{T_b, T_t\} \\
y_t & \text{when } t > T_t \text{ and } T_t < T_b \\
0 & \text{when } t > T_b \text{ and } T_b < T_t \end{cases}, \\
m_{t+1} &= \begin{cases} 
m_t + j_t - (\beta_t - \beta_{t-1})K - c_t & \text{when } t < \min\{T_b, T_t\} \\
m_t + j_t & \text{when } t = T_t \text{ and } T_t < T_b \\
m_t + j_t + c_t & \text{when } t = T_b \text{ and } T_b < T_t \\
m_t - c_t & \text{when } t \geq T_t \text{ and } T_t < T_b \end{cases}, \\
W_t &= m_t + j_t.
\end{align*}
\end{equation}

The third equality in (11) provides the cash balance of the manager’s saving account under different circumstances.

Now we define the agents’ choice problems at a time when the money has already been borrowed and the parcel of land has already been purchased. The decision problems to be faced by the manager and investor are

**Problem 7 Manager’s choice (Debt-Repurchase)** Given the sinking fund requirement on debt-repurchase over time \( \{\beta_t^*|\beta_0^* = 0, \beta_t^* \in [0, 1] \text{ and } \beta_t^* \leq \beta_{t-1}^*(1 + r) + \frac{\beta_t^*}{K}, t \geq 1\} \), the required interest payments over time \( \{i_t^* = (1 - \beta_t^*)K, t \geq 1\} \) (where \( (1 - \beta_{t-1})K \) equals total outstanding debt) and the investor’s strategy \( \sigma_i \), the manager chooses \( \sigma_m \) to maximize the expected lifetime utility of consumption and effort \( \sum_{t=1}^{\infty} \frac{c_t - e_t}{(1+r)^t} \) subject to the constraints in (10) and (11).

**Problem 8 Investor’s choice (Debt-Repurchase)** Given the sinking fund requirement on debt-repurchase over time \( \{\beta_t^*|\beta_0^* = 0, \beta_t^* \in [0, 1] \text{ and } \beta_t^* \leq \beta_{t-1}^*(1 + r) + \frac{\beta_t^*}{K}, t \geq 1\} \), the required interest payments over time \( \{i_t^* = (1 - \beta_t^*)K, t \geq 1\} \) and the manager’s strategy \( \sigma_m \) satisfying the constraints in (10), the investor chooses \( \sigma_i \) to maximize the lifetime cash flow \( \sum_{t=1}^{T_{d}} \frac{i_t + (\beta_t - \beta_{t-1})K}{(1+r)^t} + \frac{\min\{K, (1+r)(1-\beta_{d-1})K-i_d\}}{(1+r)^d} \) subject to the constraints in (11), where \( T_{d} \equiv \min\{T_b, T_t\} \).
Results on Debt Financing

We have the following result on debt financing of the project.

**Theorem 5 Equilibrium (Outside Debt with Debt-Repurchase)** In the simple proprietorship model with debt-repurchase, whether the interest payments over time $\{i_t^* = (1-\beta_{t-1})rK, t \geq 1\}$ (where $(1-\beta_{t-1})K$ equals total outstanding debt) can be maintained over time depends on how large is the effectiveness of effort $\alpha$ and whether a sinking fund is used:

(i) **Financing cannot be supported by the threat to force bankruptcy itself:** When no sinking fund is used, interest payments cannot be maintained, and in fact no equilibrium has positive interest payments along the equilibrium path.

(ii) **Financing can be supported by sinking fund:** With a sinking fund, the interest payments can be maintained, using the “good proprietorship equilibrium” below when $1 + \frac{r^2K}{\bar{e}} \leq \alpha \leq \frac{(1-r)K}{\bar{e}}$. The good proprietorship equilibrium that provides the quickest repurchase of debt is the equilibrium in which the manager is required to spend all the income on debt-repurchase:

\[
\begin{align*}
\beta_t(h^t, e^t) & = \begin{cases} 
\beta_t^* & \text{if } (\forall s < t) i_s \geq i_s^* \text{ and } \beta_s \geq \beta_s^* < 1, \\
\beta_{t-1} & \text{otherwise}
\end{cases}, \\
\beta_t(h^t, e^t) & = \begin{cases} 
\beta_t^* & \text{if } (\forall s < t) i_s \geq i_s^* \text{ and } \beta_s \geq \beta_s^* < 1, \\
\beta_{t-1} & \text{otherwise}
\end{cases}, \\
l_t(h^t, e^t) & = \begin{cases} 
0 & \text{when } (\forall s \leq t) i_s \geq i_s^* \text{ and } \beta_s \geq \beta_s^* < 1, \\
1 & \text{otherwise}
\end{cases}.
\end{align*}
\]

where

\[
\beta_t^* = \begin{cases} 
\min\{\beta_{t-1}(1+r) + \frac{\alpha^e}{K}, 1\} & \text{when } \beta_{t-1} < 1, \\
1 & \text{otherwise}
\end{cases}.
\]

**Proof:** See Appendix A.

The intuition behind the result on debt financing is similar to that behind the result on equity financing. In order to receive debt financing, a project can neither be too profitable nor be too unprofitable. In order for a project to be financed with debt, a large enough proportion of the debt needs to be retired when the manager has enough wealth to take over the project.
7 Conclusion

This paper considers the optimal choice between two different organizational forms for a project whose cash flows are observable to both the manager and investor, but not publicly verifiable. The nonverifiability of the firm’s cash flows creates an extreme form of the manager’s moral hazard problem—the manager can take the entire cash flow out of the firm if given the control over the firm’s cash flow—which can both make the financing of the project difficult to arrange and have a large impact on the allocation of the control over the output of the firm and the form of managerial compensation. We find that the two organizational forms can have very different incentive problems. For the corporation form, the employee-manager may have an incentive to expend too little effort on the project, because the manager’s income is guaranteed by a fixed-wage contract. For the proprietorship, the owner-manager has no incentive to shirk, but may have an incentive to keep the entire cash flow and default on the debt. Depending on the relative severity of the two incentive problems, either organizational form can dominate the other. The possibility of an MBO can exacerbate the incentive problems and make it impossible to prevent shirking or defaulting on the debt with only the threat of dismissal or the threat of liquidation. Certain institutions, such as sinking funds, employee share ownership and managers’ non-vested pensions, can be used to mitigate the incentive problems and support the financing of the project.

The paper has two predictions on the financing of the firm. One prediction is that the corporate form is more likely to be used by non-effort-intensive firms and the proprietorship is more likely to be used by non-capital-intensive firms. The other prediction is that it is in general more difficult to arrange equity financing than to arrange debt financing, and as a result, debt may be the preferred choice of the two when both can be used and we may see debt being used more often than equity.

Even though this paper compares debt with equity under a specific situation in which not all the managers’ decisions can be pre-specified with an enforceable contract, the main results of the paper—debt and equity induce different incentive problems and depending on the relative severity of the two incentive problems, either debt or equity can dominate the other—can hold under many other circumstances in which incomplete contracting exists. Considering the optimal choice between debt and equity or the optimal combination of debt and equity under these circumstances may be interesting for future research.
A Proofs of the Theorems

Here we provide only the proofs of important results in the theorems and only the key steps in the proofs.

Proof of Theorem 1: First, we prove (i). We first show that the manager’s strategy is the optimal response to the investor’s strategy in all subgames along the equilibrium path. Playing the proposed equilibrium strategy, the manager’s utility is $\alpha e - e r$. If the manager deviates and chooses $e_t < e$ first in period $t$, the manager will then be fired and the utility is instead $\frac{\alpha e - e_t}{1 + r} + \frac{\alpha e - e_t}{(1 + r)^t - 1} + \frac{\alpha e - e_t}{1 + r}$. The change in expected utility is given by $\frac{\alpha e - e_t}{1 + r} - \frac{\alpha e - e_t}{r(1 + r)^{t-1}} \leq 0$ by assumption $\alpha \geq 1 + r$. So deviation does not pay.

Next, we show that good corporate equilibrium strategies are Nash equilibrium strategies in all the subgames on every off-equilibrium path. First, we want to ask whether, if the manager did deviate by choosing $e_t < e$ in period $t$, the investor would choose to liquidate the firm and fire the manager. The manager’s strategy indicates that once he chooses his effort level to be below $e$, he will always choose zero effort level. Liquidating the project in period $t$ gives the investor his expected lifetime cash flow $-K + \sum_{s=1}^{t-1} \frac{rK}{(1 + r)^s} + \frac{rK + \alpha e_t}{(1 + r)^s} + \frac{K}{(1 + r)^s}$ while delaying the liquidation of the project until period $i$ for $i > t$ gives the investor his expected lifetime cash flow $-K + \sum_{s=1}^{i-1} \frac{rK}{(1 + r)^s} + \frac{\alpha e_t}{(1 + r)^s} + \sum_{s=t}^{i} \frac{rK - \alpha e_t}{(1 + r)^s} + \frac{K}{(1 + r)^s}$. The change in expected lifetime cash flow is given by $-\sum_{s=t+1}^{i} \frac{\alpha e_t}{(1 + r)^s} \leq 0$. So the investor’s optimal strategy after the manager’s deviation is to liquidate the firm immediately.

Next, we want to ask whether the manager should choose zero effort level in the subgame in which the manager did deviate by choosing $e_t < e$ in period $t$, but the investor did not liquidate the project and fire the manager in the same period. The investor’s strategy indicates that once the manager chooses $e_t < e$, the investor will always choose to liquidate the project. Expecting the investor to liquidate the project at the end of the period, the manager’s optimal decision is to play his candidate bad equilibrium strategy: choose zero effort level and just collect the wage.

Now we prove (ii). We prove this result by first assuming that there is an equilibrium with positive effort (along the equilibrium path) and then showing that this assumption leads to a contradiction. Suppose that there exists an equilibrium with a constant wage and a positive constant effort over time, i.e. there exist a $w \geq 0$ and an $e \in (0, \bar{e}]$ such that for $w_t = w$ and $e_t = e \forall t \geq 1$.
the following strategies

\begin{align}
\text{(14)} & \quad e_t(h^t) = \begin{cases} 
e & \text{if } (\forall s < t)e_s = e \\
0 & \text{otherwise} \end{cases} \\
& \quad l_t(h^t, e_t) = \begin{cases} 0 & \text{when } (\forall s \leq t)e_s = e \\
1 & \text{otherwise} \end{cases}
\end{align}

are subgame perfect Nash equilibrium strategies. Given the investor’s strategy, the manager’s utility from the proposed equilibrium strategy is again \( w-e \). The manager’s utility from a deviation from the proposed equilibrium strategy at time \( t \) by choosing \( e'_t < e \) is \( w-e \left( 1 - \frac{1}{(1+r)^t} \right) + \frac{w-e_t'}{(1+r)^t} \). The change in expected utility is given by

\begin{align}
\text{(15)} \quad & \quad \frac{w-e'_t}{(1+r)^t} - \frac{w-e}{r(1+r)^{t-1}} = \frac{r(w-e'_t)-(w-e)(1+r)}{r(1+r)^t}.
\end{align}

The assumption that the strategies in (14) are equilibrium strategies implies that the right-hand-side of (15) is nonpositive \( \forall e'_t < e \). Specifically, the right-hand-side of (15) should be nonpositive when \( e'_t = 0 \), i.e.,

\begin{align}
\text{(16)} \quad & \quad \left( \frac{r(w-e'_t)-(w-e)(1+r)}{r(1+r)^t} \right) \bigg|_{e'_t=0} = \frac{e(1+r) - w}{r(1+r)^t} \leq 0.
\end{align}

From (16), we have \( w \geq (1+r)e \). With this lower bound on the manager’s wage, we can compute the equityholder’s cash flow in each period

\begin{align}
\text{(17)} \quad & \quad y_t - w = rK + \alpha e - w \leq rK + \alpha e - (1+r)e < rK
\end{align}

by assumption that \( \alpha < (1+r) \) and \( e > 0 \). (17) implies that the equityholder cannot breakeven when providing the financing of the project in this (assumed) equilibrium. So such an equilibrium does not exist.

**Proof of Theorem 2:** Because of the similarity between this proof and the proof of Theorem 1, we provide only the proof of (i) and (ii) and only the key steps in the proof.

First, we prove (i). Here we prove only that the manager’s stratgety is the optimal response to the investor’s strategy in all subgames along the equilibrium
path. Given the investor’s strategy, the manager’s utility from the proposed equilibrium strategy is $\frac{\alpha e - e}{r}$. The manager’s utility from a deviation from the proposed equilibrium strategy at time $t$ by choosing $i_t < rK$ is $\sum_{s=1}^{t} \frac{(\alpha-1)e}{(1+r)^s} + \frac{rK-i_t}{(1+r)^t}$. The change in expected utility is given by $\frac{rK-i_t}{(1+r)^t} - \frac{(\alpha-1)e}{r(1+r)^t} \leq -\frac{i_t}{(1+r)^t} \leq 0$ by assumption $\alpha \geq 1 + \frac{r^2K}{e}$. So the deviation does not pay.

Next, we prove (ii). We prove this result by first assuming that there is an equilibrium with positive interest payments (along the equilibrium path) and then showing that this assumption leads to a contradiction. Suppose that there exists an equilibrium with positive constant interest payments over time, i.e. there exists an $i \in (0, rK + \alpha e]$ such that for $i_t = i \ \forall \ t \geq 1$, the following strategies

\begin{equation}
\begin{align*}
i_t(h^t) &= \begin{cases} 
i & \text{if } (\forall s < t) i_s \geq i \\
0 & \text{otherwise}
\end{cases} \\
l_t(h^t, i_t) &= \begin{cases} 
0 & \text{when } (\forall s \leq t) i_s \geq i \\
1 & \text{otherwise}
\end{cases}
\end{align*}
\end{equation}

are subgame perfect Nash equilibrium strategies. Given the investor’s strategy, the manager’s utility from the proposed equilibrium strategy is $\sum_{s=1}^{\infty} \frac{rK-i+(\alpha-1)e}{(1+r)^s}$. The manager’s utility from a deviation from the proposed equilibrium strategy at time $t$ by choosing $i'_t < i$ is $\sum_{s=1}^{t} \frac{rK-i+(\alpha-1)e}{(1+r)^s} + \frac{i-i'_t}{(1+r)^t}$. The change in expected utility is given by

\begin{equation}
\frac{i - i'_t}{(1+r)^t} - \frac{rK-i+(\alpha-1)e}{r(1+r)^t} = \frac{r(i - i'_t) - (rK - i + (\alpha-1)e)}{r(1+r)^t}.
\end{equation}

The assumption that the strategies in (18) are equilibrium strategies implies that the right-hand-side of (19) is nonpositive $\forall i'_t < i$. Specifically, the right-hand-side of (19) should be nonpositive when $i'_t = 0$, i.e.,

\begin{equation}
\left. \frac{r(i - i'_t) - (rK - i + (\alpha-1)e)}{r(1+r)^t} \right|_{i'_t=0} = \frac{ri - rK + i - (\alpha-1)e}{r(1+r)^t} \leq 0.
\end{equation}

From (20), we have

\begin{equation}
i \leq \frac{rK + (\alpha-1)e}{1+r} < \frac{rK + r^2K}{1+r} = rK
\end{equation}
by assumption \( \alpha < 1 + \frac{rK}{r} \). (21) implies that the debtholder cannot breakeven when providing the financing of the project in this (assumed) equilibrium. So such an equilibrium does not exist.

**Proof of Theorem 4:** We provide only the proof of (i) and (ii) and only the key steps in the proof.

First, we prove (i). It suffices to show that the equityholder’s strategy for dismissal in the good corporate equilibrium in Theorem 1 itself cannot deter the manager from shirking. One way to prove this is to show that given the equityholder’s strategy for dismissal in (9), the manager is better off playing the following strategy on effort and consumption

\[
(22) \quad e_t(h^t) = \begin{cases} 
\bar{e} & \text{when } \beta_{t-1} = 0 \text{ and } W_t < K \\
0 & \text{when } \beta_{t-1} = 0 \text{ and } W_t \geq K \\
\bar{e} & \text{when } \beta_{t-1} = 1 
\end{cases}, \\

\[
\eta_t(h^t) = \begin{cases} 
0 & \text{when } \beta_{t-1} = 0 \\
\leq W_t & \text{when } \beta_{t-1} = 1
\end{cases},
\]

which involves shirking (when the manager is able to take over the firm) and taking over the firm through MBO, rather than the strategy (on effort and consumption) in (9).

Given the equityholder’s strategy in (9), the manager’s utility from playing the strategy in (9) is \( \frac{\alpha e - e}{r} \). If instead playing the strategy in (22), the manager gets \( (\alpha - 1) e + \frac{e (1 + \sigma t)}{r} \) where \( T_m' \equiv \inf \{ t | \beta_t = 1 \text{ and } W_t \geq K, \ t \geq 1 \} \). The gain for the manager equals \( \frac{\sigma - \bar{e}}{(1+r)^{T_m'}} > 0 \).

Next, we prove (ii) and we prove only that the manager’s stratgety is the optimal response to the investor’s strategy in all subgames along the equilibrium path. Given the equityholder’s strategy in (9), the manager’s utility from playing the strategy in (9) is \( \frac{\alpha e - \bar{e}}{r} \). Now we consider a deviation from the strategy in (9) is \( \frac{\alpha e - \bar{e}}{r} \). We consider a deviation from the strategy in (9). Suppose the manager chooses \( e_t < \bar{e} \) in period \( t \) when \( \beta_{t-1} < 1 \) and \( W_t < K \). Then the manager’s utility from playing this new strategy is \( \frac{\alpha e}{(1+r)^{T_m'}} + \frac{(\alpha - 1) e}{r} \left( 1 - \frac{1}{(1+r)^{T_m'-1}} \right) + \frac{\alpha e_{t-1} (e_t - \bar{e})}{(1+r)^{T_m'}} \). The change in expected utility is given by \( \frac{\sigma - \bar{e}}{(1+r)^{T_m'}} (1 + r - \alpha) + \frac{\alpha e_{t-1} (e_t - \bar{e})}{(1+r)^{T_m'}} \leq 0 \) by assumption \( \alpha \geq 1 + r \). So the deviation does not pay.

\[ ^{13} \]Since the investor does not compensate the manager in the company’s shares before the investor takes over the firm, the manager’s ownership of the firm is zero before the MBO and one after the MBO.
Now we consider the situation in which the manager chooses \( e_t < \bar{\pi} \) in period \( t \) when \( \beta_{t-1} < 1 \) and \( W_t \geq K \). The manager’s utility from playing this new strategy is \( \sum_{s=1}^{\infty} \frac{(\alpha-1)\bar{\pi}}{(1+r)^s} + \frac{(1-\alpha)\beta_{t-1}(\bar{\pi}-\bar{\alpha})}{(1+r)^t} \). The change in expected utility is \( \frac{(1-\alpha)\beta_{t-1}(\bar{\pi}-\bar{\alpha})}{(1+r)^t} \). It is nonpositive if and only if \( \beta_{t-1} \geq \frac{1}{\alpha} \). To deter the manager’s shirking and taking over the firm \( \forall t \geq T_{mbo} \), we need \( \beta_{t-1} \geq \frac{1}{\alpha}, \forall t \geq T_{mbo} \).

**Proof of Theorem 5:** We provide only the proof of (ii) and only the key steps in the proof.

Now we prove (ii) and we prove only that the manager’s strategy is the optimal response to the investor’s strategy in all subgames along the equilibrium path. First, it is quite clear that we need to have \( \alpha \geq 1 + \frac{r^2 K}{\bar{\pi}} \), otherwise the manager can benefit from defaulting on the debt in the first period. It is also quite clear that we need to have \( \alpha < \frac{(1-r)K}{\bar{\pi}} \), otherwise the manager can default on the debt in the first period and then take over the project through debt-repurchase. Given the debtholder’s strategy in (12) and (13), the manager’s utility from playing the strategy in (12) and (13) is \( \frac{\alpha e}{r} \). Now we consider a deviation (i.e. \( i_t < rK \) and \( \beta_t = \beta_{t-1} \)) in period \( t \) when \( \beta_{t-1} < 1 \) and \( W_t \geq \min\{K, (1-\beta_{t-1})(1+r)K\} \). Then the manager’s utility from playing this new strategy is \( \frac{rK+(\alpha-1)\bar{\pi}-i_t}{(1+r)^t} \left( 1 - \frac{1}{(1+r)^{t-1}} \right) + \max\left\{ \frac{\alpha e}{r} \left( 1 - \frac{1}{(1+r)^{t-1}} \right) - \frac{rK}{(1+r)^t}, 0 \right\} \). The change in expected utility is given by \( -\frac{(\alpha-1)\bar{\pi}}{r(1+r)^t} - \frac{i_t}{(1+r)^t} \leq 0 \) when \( \frac{\alpha e}{r} \left( 1 - \frac{1}{(1+r)^{t-1}} \right) > \frac{rK}{(1+r)^t} \), and \( \frac{\alpha e}{r} \left( 1 - \frac{1}{(1+r)^{t-1}} \right) \leq 0 \) (by assumption \( \alpha \geq 1 + \frac{r^2 K}{\bar{\pi}} \)) when \( \frac{\alpha e}{r} \left( 1 - \frac{1}{(1+r)^{t-1}} \right) \leq \frac{rK}{(1+r)^t} \).

Now we consider the same deviation (i.e. \( i_t < rK \) and \( \beta_t = \beta_{t-1} \)) from the strategy in (12) and (13) when \( \beta_{t-1} < 1 \) and \( W_t \geq \min\{K, (1-\beta_{t-1})(1+r)K\} \). Since \( W_t = rK + \alpha e - i_t < K \) by assumption \( \alpha < \frac{(1-r)K}{\bar{\pi}} \), the manager is going to be able to take over the firm through debt-repurchase only if \( (1-\beta_{t-1})(1+r)K < K \) (to be exactly, only if \( (1-\beta_{t-1})(1+r)K \leq rK + \alpha e < K \)). The manager’s utility from playing this new strategy is \( \sum_{s=1}^{+\infty} \frac{(\alpha-1)\bar{\pi}}{(1+r)^s} - \frac{i_t}{(1+r)^s} = \frac{(\alpha-1)\bar{\pi}}{r} - \frac{i_t}{(1+r)^s} \). The change in expected utility is given by \( -\frac{i_t}{(1+r)^t} \leq 0 \). So the deviation does not pay.
References


